Clusters in Nuclear Matter and Heavy Nuclei

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TECHNISCHE UNIVERSITÄT DARMSTADT

Seminar

School of Physics
Aristotle University
Thessaloniki, Greece
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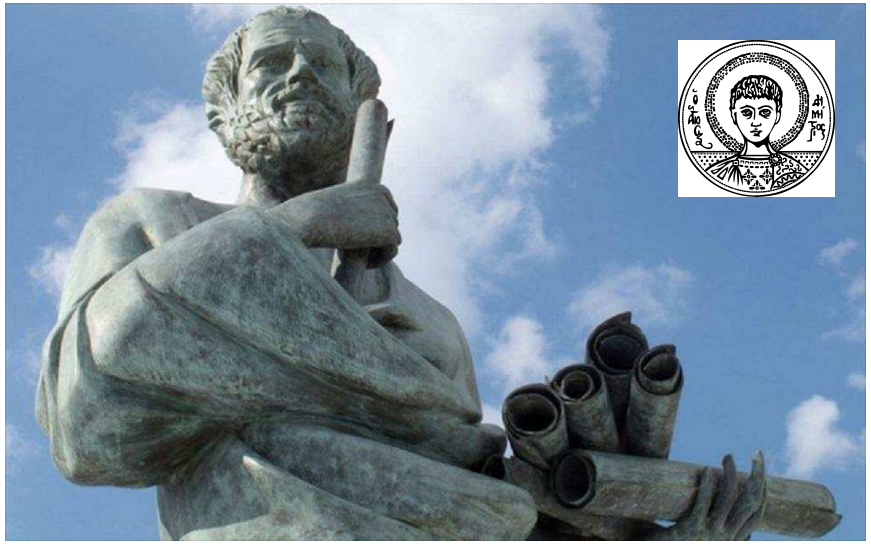

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Outline

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- Introduction
- Generalized Relativistic Density Functional
- Correlations and Clusters
- Compact Star Matter
- Surface Properties of Heavy Nuclei
- Correlations above Nuclear Saturation Density
- Conclusions

Introduction

History of the Cosmos

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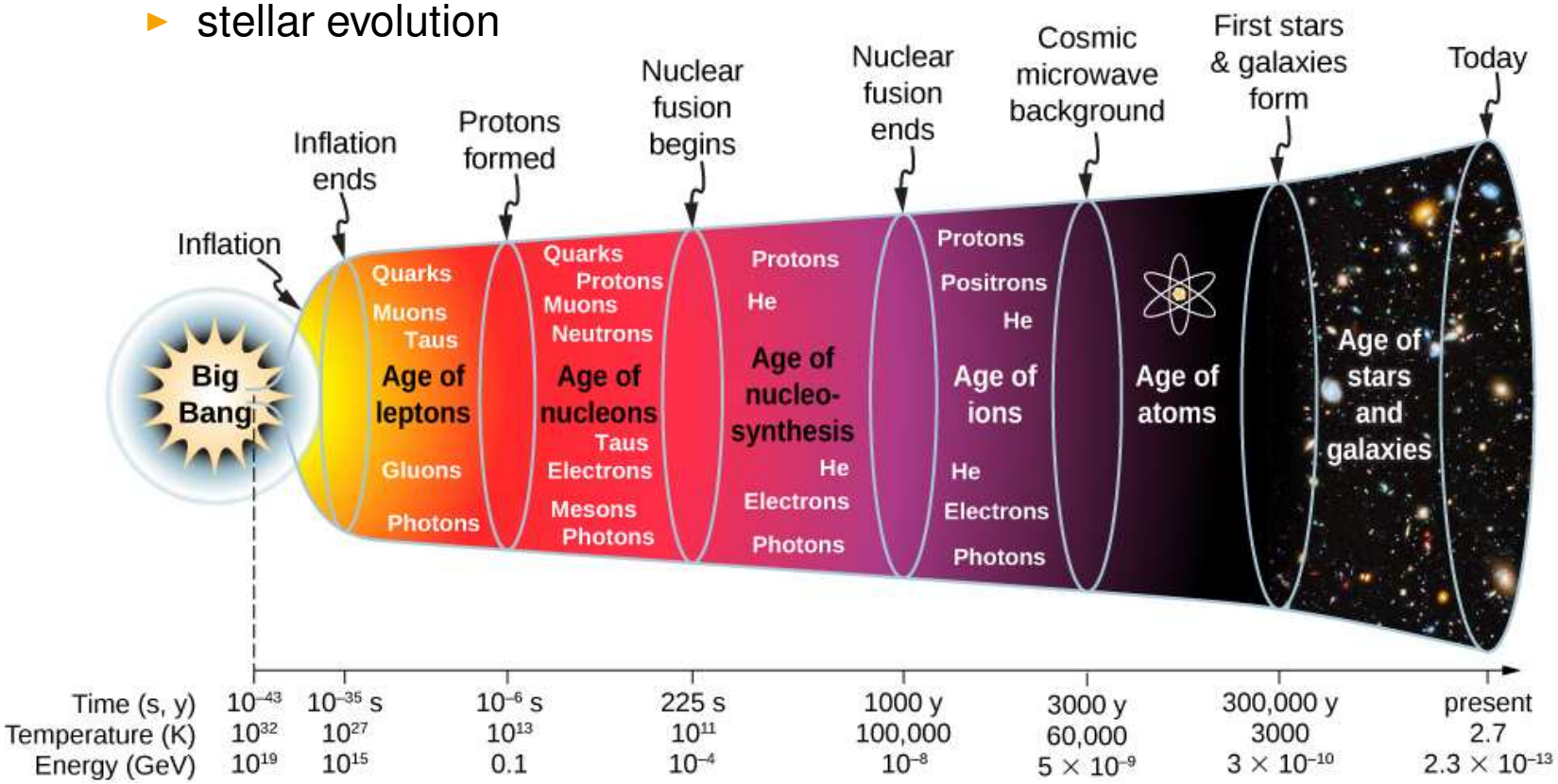
- origin of chemical elements
  - primordial nucleosynthesis
  - stellar evolution



source: phys.libretexts.org

Stellar Evolution

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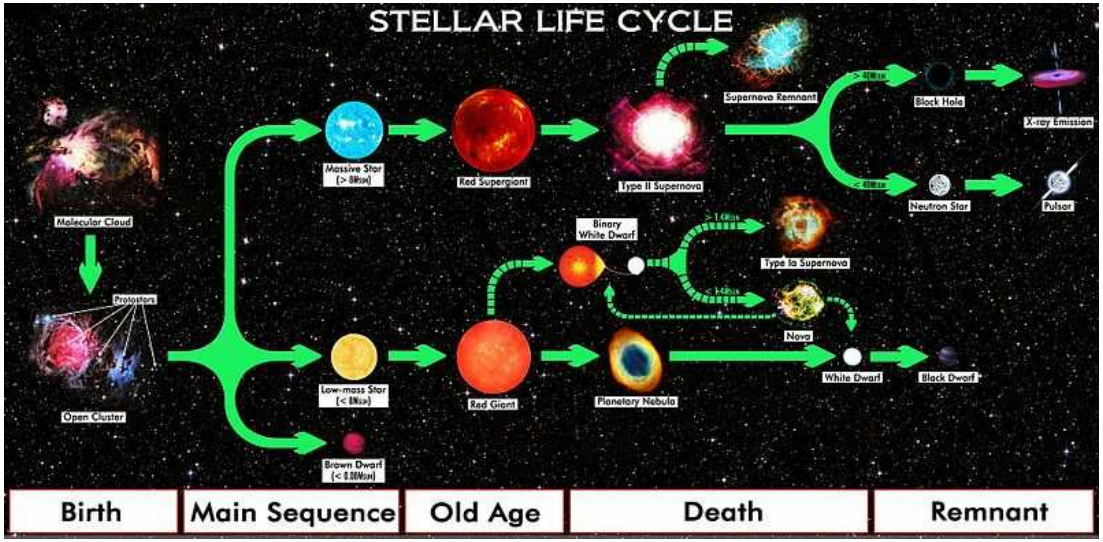
- quiet stellar burning
  - main sequence stars, red giants, . . .
    $\Rightarrow$ elements up to iron group, s process elements
- violent/explosive events
  - supernovae of different types, neutron star mergers
    $\Rightarrow$ heavy elements beyond iron



source: R. N. Bailey, en.wikipedia.org

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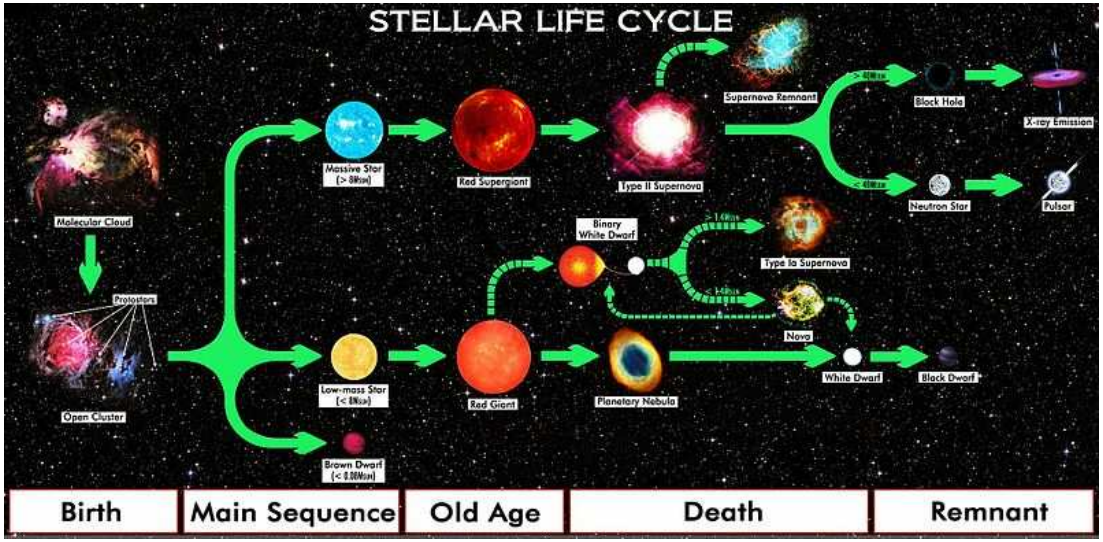
What is needed in astrophysical simulations of these events?



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Astrophysical Simulations

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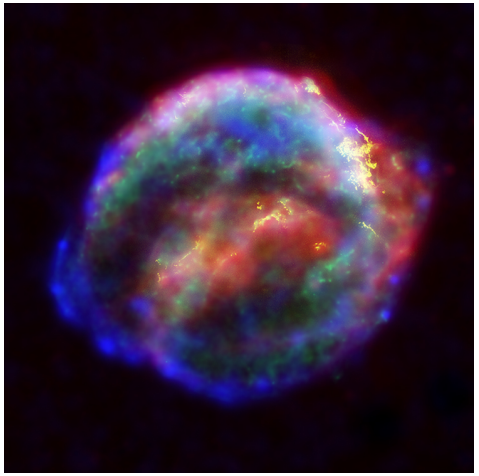
- late stages in evolution of heavy stars
  - core-collapse supernovae
    $\Rightarrow$ formation of neutron stars
  - merger of neutron stars

X-ray: NASA/CXC/J.Hester (ASU)
Optical: NASA/ESA/J.Hester & A.Loll (ASU)



NASA/ESA/R.Sankrit & W.Blair (Johns Hopkins Univ.)

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- late stages in evolution of heavy stars
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- required **physics input** in models
  - general relativity
  - hydrodynamics
  - nuclear reaction rates
  - neutrino physics
  - **properties of dense nuclear matter**
    $\Rightarrow$ **equation of state (EoS)**

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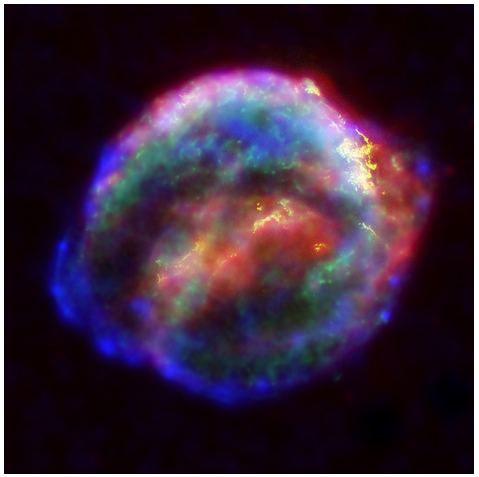

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- thermodynamic conditions?

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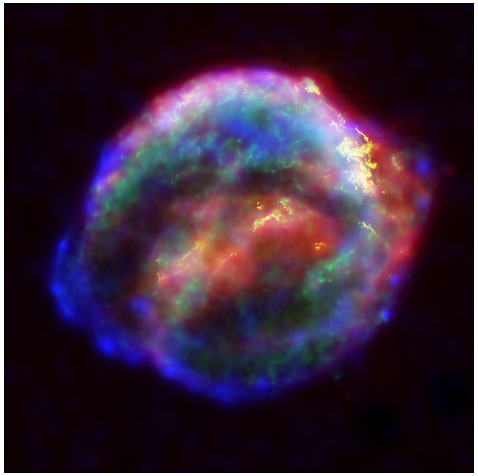

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Equation of State (EoS) in Astrophysics

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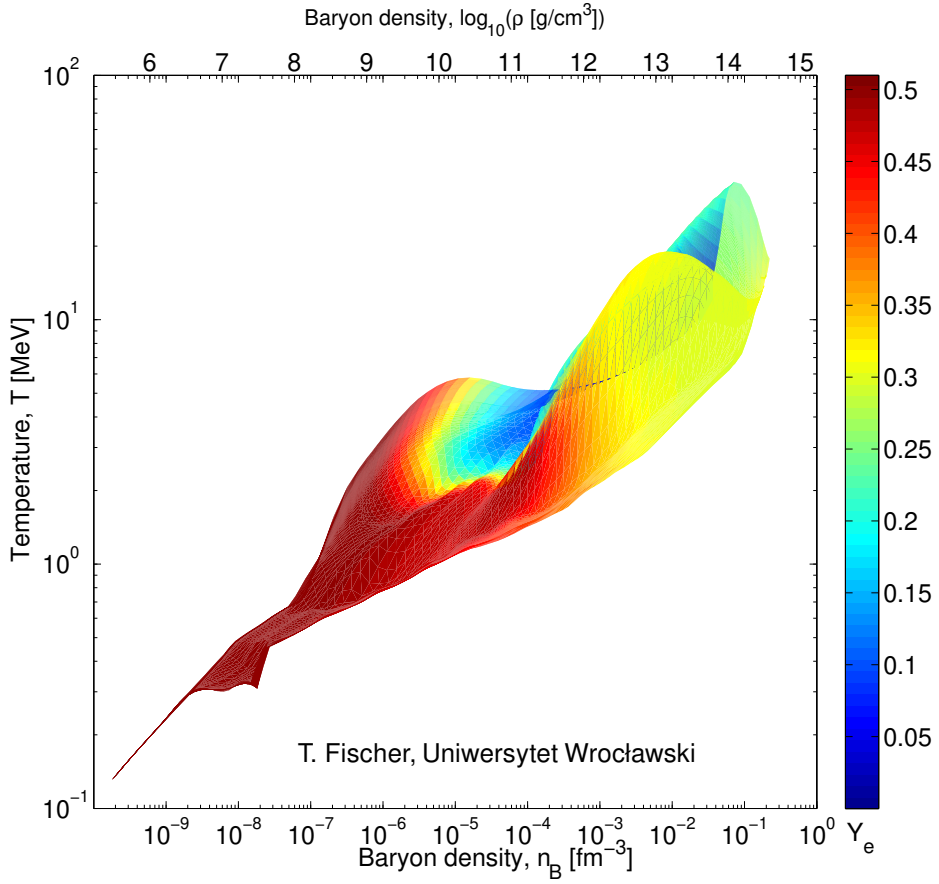
variables

- density:
  $10^{-10} \lesssim \varrho/\varrho_{\text{sat}} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\text{sat}} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\text{sat}} = \varrho_{\text{sat}}/m_{\text{nuc}} \approx 0.15$ fm^{-3})
- temperature:
  0 MeV $\leq k_B T \lesssim$ 50 MeV ($\hat{=}$ $5.8 \cdot 10^{11}$ K)
- electron fraction:
  $0 \leq Y_e = n_e/n_B \lesssim 0.6$
  with electron (baryon) density n_e (n_B)

simulation of core-collapse supernova



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$\Rightarrow$ **global, multi-purpose EoS** required

EoS database: `compose.obspm.fr`

EoS review: M. Oertel et al.,
Rev. Mod. Phys. 89 (2017) 015007

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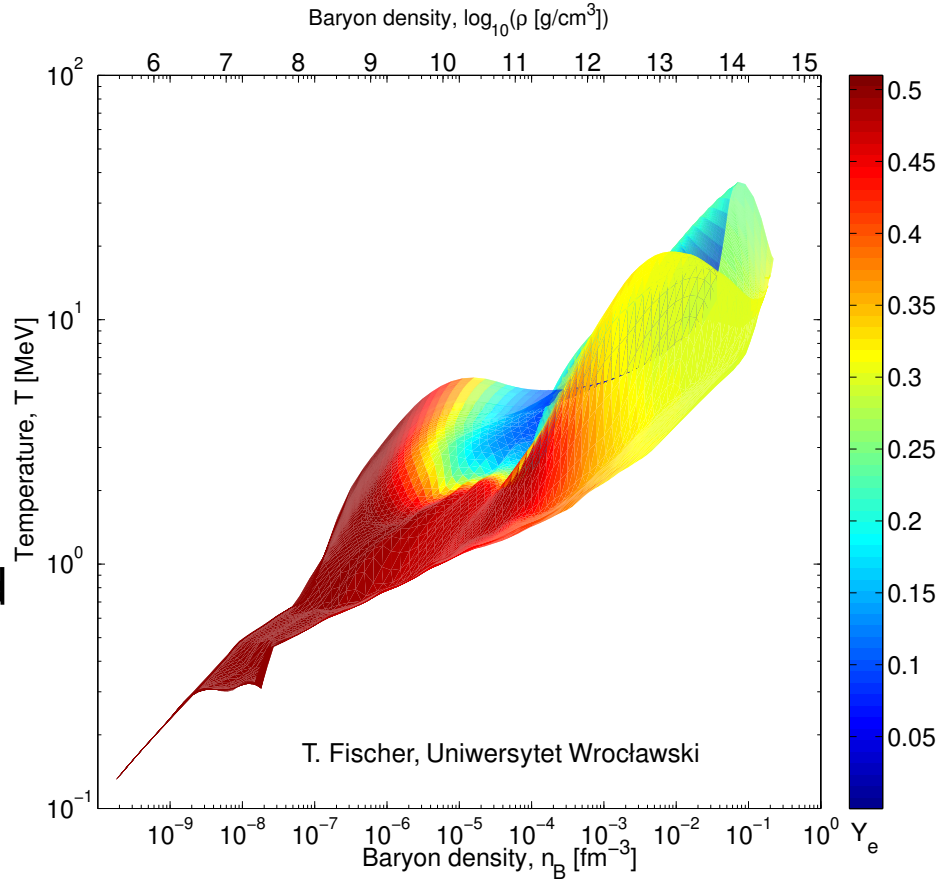

Theoretical Approaches

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- **hadronic 'ab-initio' methods with realistic interactions**
  - interactions: potential models, meson-exchange, chiral forces, RG evolved, ...
    (Argonne, Urbana, Tucson-Melbourne, Nijmegen, Paris, Bonn, ...)
    $\Rightarrow$ two-body NN interaction (in vacuum) well constrained by experiment,
    three-body forces less, large uncertainties for YN, YY, ...
  - many-body methods: BHF/DBHF, SCGF, CBF, VMC, GFMC, AFDMC, ...
- **QCD-based/inspired descriptions**
- **effective field theories (EFT)**

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  - many EoS for neutron matter & neutron star matter, but no global EoS
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$\Rightarrow$ **phenomenological models** for global EoS

EoS for Astrophysical Applications

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- **constituents:** mostly considered are
  nucleons, nuclei (light/heavy/representative), leptons, photons, ...
- **models:** often combination of different approaches
  (Skyrme/Gogny/relativistic mean-field models, NSE, virial EoS,
  density functionals, classical/quantum molecular dynamics, ...)

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  (Skyrme/Gogny/relativistic mean-field models, NSE, virial EoS,
  density functionals, classical/quantum molecular dynamics, ...)
- **global EoSs used in astrophysical simulations:**
  - H&W: W. Hillebrandt, K. Nomoto, R.G. Wolff, A&A 133 (1984) 175
  - LS180/220/375: J.M. Lattimer, F.D. Swesty, NPA 535 (1991) 331
  - STOS (TM1): H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi, NPA 637 (1998) 435, PTP 100 (1998) 1013
  - HS (TM1,TMA,FSUgold,NL3,DD2,IUFSU): M. Hempel, J. Schaffner-Bielich, NPA 837 (2010) 210
  - SHT (NL3): G. Shen, C.J. Horowitz, S. Teige, PRC 82 (2010) 015806, 045802, PRC 83 (2011) 035802
  - SHO (FSU1.7, FSU2.1): G. Shen, C.J. Horowitz, E. O'Connor, PRC 83 (2011) 065808
  - SFHo/SFHx: A.W. Steiner, M. Hempel, T. Fischer, ApJ 774 (2013) 17
  - recently many more, also with additional degrees of freedom (hyperons, quarks, ...)

$\Rightarrow$ here: **generalized relativistic density functional**

Generalized Relativistic Density Functional

Density Functionals for Nuclei and Nuclear Matter

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- various types (nucleons, hyperons, other baryons as degrees of freedom)
  - nonrelativistic or relativistic/covariant
  - often derived from mean-field models in different approximations (Hartree, Hartree-Fock, Hartree-Fock-Bogoliubov)

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- examples
  - Skyrme Hartree-Fock models
    - two-body interaction: zero-range with expansion in momentum up to second order
    - three-body interaction: zero-range, repulsive
  - Gogny Hartree-Fock models
    - two-body interaction: finite-range of Gaussian form (two terms)
    - three-body interaction: as in Skyrme
  - relativistic models
    - field-theoretical approach, mean-field approximation
    - interaction by meson exchange (σ, ω, ρ, ...)
    - medium effects:
      - nonlinear models (selfcoupling of mesons)
      - density dependent couplings

Relativistic Density Functional

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- relativistic mean-field model with density dependent meson-nucleon couplings $\Rightarrow$ grandcanonical ensemble
- **degrees of freedom**
  - nucleons
    $\Rightarrow$ quasiparticles with effective mass $m_i^* = m_i - S_i$
    and effective chemical potential $\mu_i^* = \mu_i - V_i$
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  - mesons (σ, ω, ρ)
- **effective in-medium interaction**
  - minimal coupling of nucleons to mesons
    (parametrization DD2 with realistic nuclear matter parameters)

$\Rightarrow$ scalar (S_i) and vector (V_i) potentials with rearrangement contributions

$\Rightarrow$ thermodynamic consistency

Parameters

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- **masses**
  - nucleons: experimental values
  - mesons:
    - ω, ρ, δ: fixed, close to experimental values
    - σ: variable, free parameter
- **couplings**
  - nucleon-meson couplings
    - couplings at reference density $\Gamma_m(\varrho_{ref})$
    - parameters for functional dependence on density

$\Rightarrow$ approx. 10 free parameters
(determined from fit to properties of finite nuclei)

Parametrisation DD2

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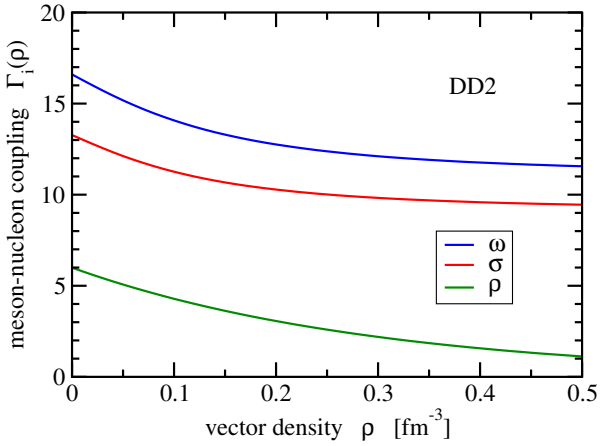
- fitted to properties of finite nuclei
  (S. Typel et al., PRC 81 (2010) 015803)
- very reasonable nuclear matter parameters
  (n_{sat} = 0.149 fm^{-3}, E_{sat} = $-$16.02 MeV,
  K = 242.7 MeV, J = 31.67 MeV, L = 55.04 MeV)



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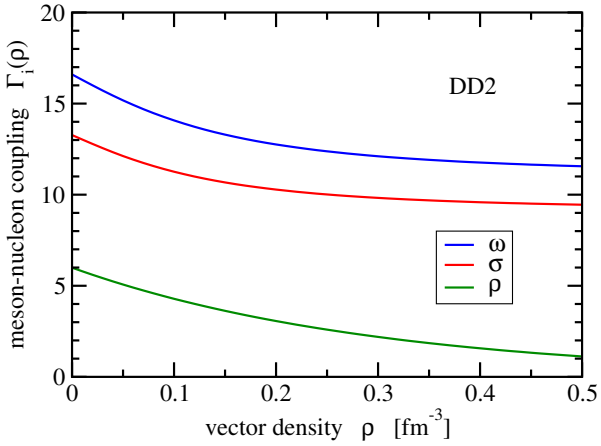
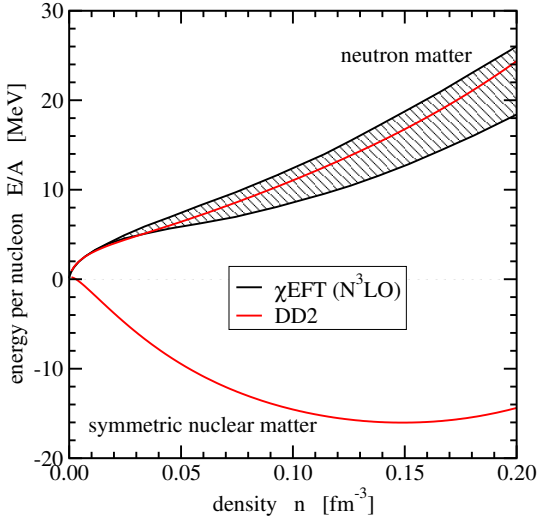
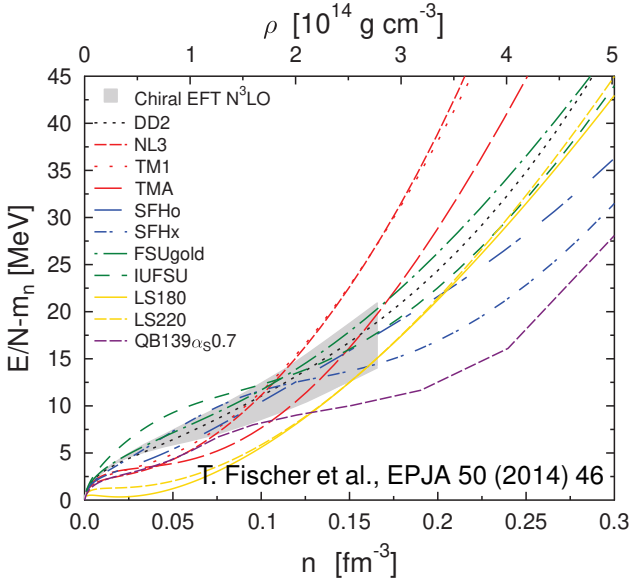
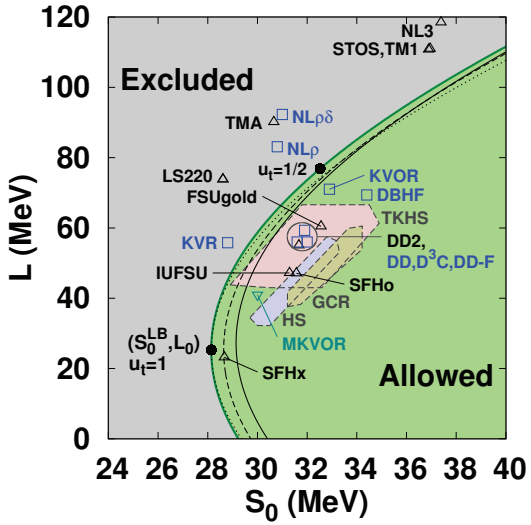
- neutron matter EoS consistent with χEFT(N^{3}LO)
  (I. Tews et al., PRL 110 (2013) 032504, T. Krüger et al., PRC 88 (2013) 025802)
- consistent with unitary gas constraint
  (I. Tews et al., ApJ 848 (2017) 105)









Generalisation of Relativistic Density Functional

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- **homogeneous nuclear matter**
  - idealized system
  - only strong interaction
  - simplified description with quasi-particles
  - no explicit correlations

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- **homogeneous nuclear matter**
  - idealized system
  - only strong interaction
  - simplified description with quasi-particles
  - no explicit correlations
- **stellar matter**
  - extended set of particle species (including antiparticles)
    - nucleons, electrons, muons, photons, hyperons (optional), ...
    - light nuclei (^{2}H, ^{3}H, ^{3}He, ^{4}He) and heavy nuclei ($A > 4$):
      vacuum bindung energies from mass tables (experiment if available)
    - two-nucleon scattering states
      $\Rightarrow$ consistency with virial EoS at low densities
    - excited states of nuclei:
      temperature dependent degeneracy factors with density of states
    - medium dependence of particle properties:
      quasiparticles (coupling to mesons, cluster mass shifts)
  - consider strong and electromagnetic interactions

Correlations and Clusters

Correlations and Composite Particles

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- interacting many-body system $\Rightarrow$ **many-body correlations**
  - at lowest densities: only two-body correlations relevant
  - with increasing density: three-, four-, many-body correlations
    $\Rightarrow$ formation of many-body bound states: **nuclei = clusters**
  - with increasing temperature: competition with **entropy**

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- **composite particles**
  - at high densities: action of **Pauli principle**
    $\Rightarrow$ blocking of states
    $\Rightarrow$ suppression of correlations
    $\Rightarrow$ dissolution of clusters
  - theoretical description?

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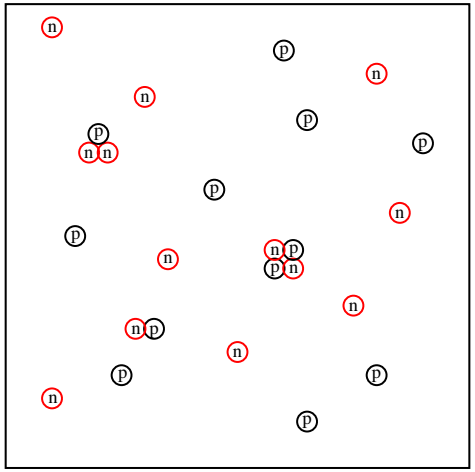
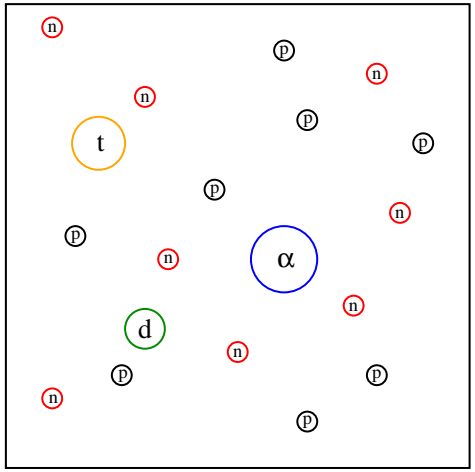
  - theoretical description?
- physical versus chemical picture





Description at Low Densities I

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- **finite temperature, exact limit $\Rightarrow$ virial equation of state (VEOS)**

  (E. Beth and G. Uhlenbeck, Physica 3(1936) 729, Physica 4 (1937) 915;
  C. J. Horowitz and A. Schwenk, NPA 776 (2006) 55)

  - expansion of pressure in powers of fugacities $z_i = \exp(\mu_i/T)$

$$p = TV\left(\sum_i \frac{g_i}{\lambda_i^3} z_i + \sum_{ij} \frac{b_{ij}}{\lambda_i^{3/2}\lambda_j^{3/2}} z_i z_j + \ldots\right) \quad \text{with thermal wavelength} \quad \lambda_i = \left[2\pi/(m_i T)\right]^{1/2}$$

  and virial coefficients $g_i, b_{ij}, \ldots \Rightarrow$ limitation $n_i \lambda_i^{-3} \ll 1$

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  - only two-body correlations relevant at lowest densities, encoded in

    $$b_{ij} = \frac{1+\delta_{ij}}{2} \frac{\lambda_i^{3/2}\lambda_j^{3/2}}{\lambda_{ij}^3} \int dE \, \exp\left(-\frac{E}{T}\right) D_{ij}(E) \pm \delta_{ij} \frac{g_i}{2^{5/2}} \qquad \lambda_{ij} = \{2\pi/[(m_i+m_j)T]\}^{1/2}$$

    with 'density of states' $D_{ij}(E) = \sum_k g_k^{(ij)} \delta(E - E_k^{(ik)}) + \sum_l \frac{g_l^{(ij)}}{\pi} \frac{d\delta_l^{(ij)}}{dE}$

    $\Rightarrow$ contribution from ground state and continuum,
    depends only on experimental data: binding energies $E_k^{(ik)}$, phase shifts $\delta_l^{(ij)}$
    (not independent! Levinson theorem)

Description at Low Densities II

TECHNISCHE UNIVERSITÄT DARMSTADT

- **simplification of VEOS**
  $\Rightarrow$ nuclear statistical equilibrium (NSE)
  - consider nucleons and all nuclei (ground and excited states)
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  $\Rightarrow$ **nuclear statistical equilibrium (NSE)**
  - consider nucleons and all nuclei (ground and excited states)
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- **extension of VEOS**
  $\Rightarrow$ **generalized (cluster) Beth-Uhlenbeck approach**

  (G. Röpke, L. Münchow, and H. Schulz, NPA 379 (1982) 536,
  M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. 202 (1990) 57,
  G. Röpke, N.-U. Bastian et al., NPA 897 (2013) 70)
  - quantum statistical description with thermodynamic Green's functions
  - part of interaction included in self-energies of quasiparticles
  - modified second virial coefficient
    $\Rightarrow$ dependence on particle-pair momentum,
    correction factor in continuum contribution

$\Rightarrow$ suppression of cluster formation with increasing density

Cluster Formation and Dissolution

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- example: deuteron as two-body correlation
  - n-p-d system, no interactions
  - no deuteron suppression at high densities in NSE or standard VEOS

Cluster Formation and Dissolution

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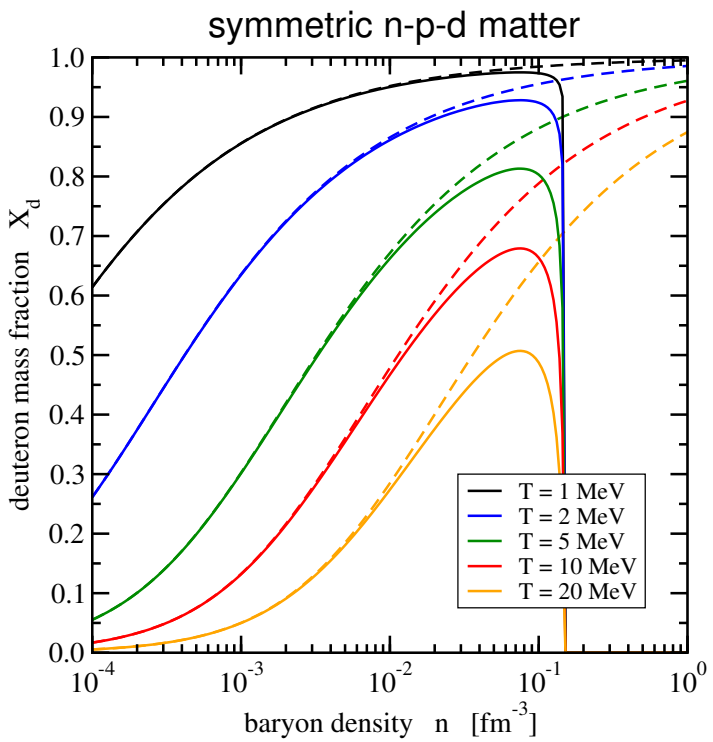
- example: deuteron as two-body correlation
  - n-p-d system, no interactions
  - no deuteron suppression at high densities in NSE or standard VEOS
- theoretical approaches for cluster suppression
  - geometric picture (finite size of particles) $\Rightarrow$ **excluded-volume mechanism**
    - applications to compact star matter
      (M. Hempel and J. Schaffner-Bielich, NPA 837 (2010) 210;
      S. Banik et al., ApJ. Suppl. 214 (2014) 22;
      T. Fischer et al., EPJ A 50 (2014) 46; M. Hempel, PRC 91 (2015) 055897)
    - generalized formulation, different interpretation
      (S. Typel, EPJ A 52 (2016) 16)



Cluster Formation and Dissolution

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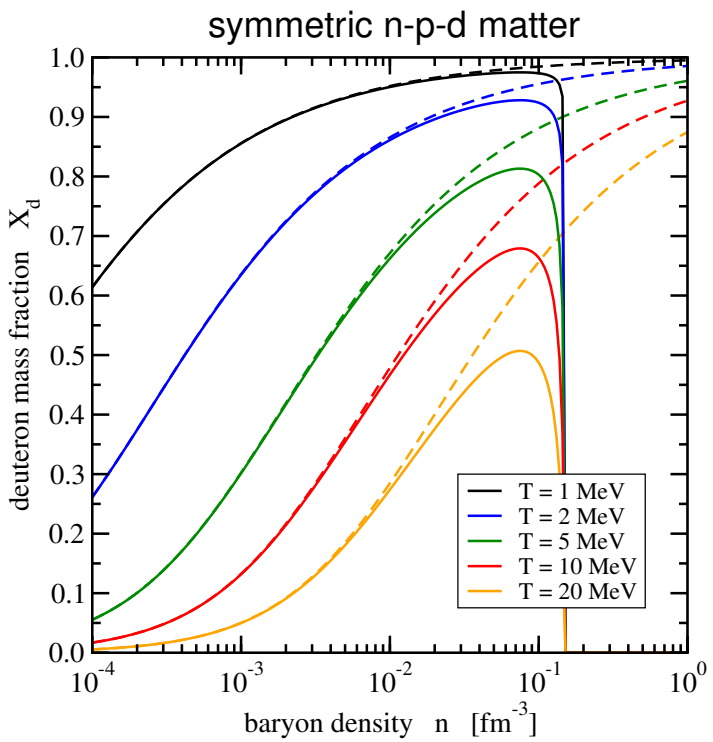
- example: deuteron as two-body correlation
  - n-p-d system, no interactions
  - no deuteron suppression at high densities in NSE or standard VEOS
- theoretical approaches for cluster suppression
  - geometric picture (finite size of particles) $\Rightarrow$ **excluded-volume mechanism**
    - applications to compact star matter
      (M. Hempel and J. Schaffner-Bielich, NPA 837 (2010) 210;
      S. Banik et al., ApJ. Suppl. 214 (2014) 22;
      T. Fischer et al., EPJ A 50 (2014) 46; M. Hempel, PRC 91 (2015) 055897)
    - generalized formulation, different interpretation
      (S. Typel, EPJ A 52 (2016) 16)
  - medium modification of cluster properties $\Rightarrow$ **mass shifts**
    - action of Pauli principle $\Rightarrow$ blocking of states
    - density, temperature, momentum dependence



Mass Shifts I

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- **concept applies to composite particles: clusters**
  - light and heavy nuclei
  - nucleon-nucleon correlations in continuum
    $\Rightarrow$ medium dependent resonances
- **effective change of masses/binding energies**

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- **effective change of masses/binding energies**
- **two major contributions** $\Delta m_i = \Delta m_i^{\text{strong}} + \Delta m_i^{\text{Coul}}$
  - strong shift $\Delta m_i^{\text{strong}} = \Delta m_i^{\text{meson}} + \Delta m_i^{\text{Pauli}}$
    - effects of strong interaction (coupling to mesons)
    - Pauli exclusion principle: blocking of states in the medium
      $\Rightarrow$ reduction of binding energies
      $\Rightarrow$ cluster dissolution at high densities: Mott effect
      $\Rightarrow$ replaces traditional excluded-volume mechanism
  - electromagnetic shift Δm_i^{Coul} (in stellar matter)
    - electron screening of Coulomb field $\Rightarrow$ increase of binding energies

$\Rightarrow$ rearrangement contribution in density functional

Mass Shifts II

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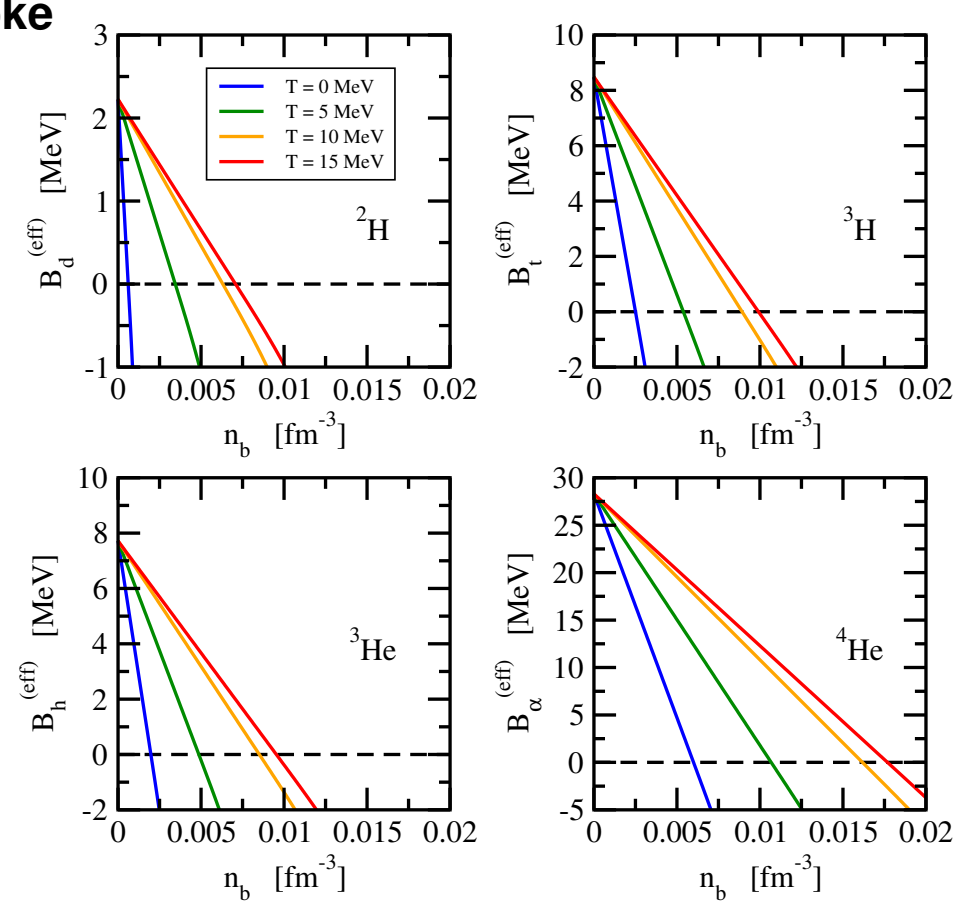
- light nuclei and NN scattering states
  - **parametrisation from Gerd Röpke**
    simplified and modified for high densities and temperatures
  - scattering states:
    mass shifts as for deuteron
  - dependence of $\Delta m_i^{\text{Pauli}}$ on temperature and effective density
    $n_i^{\text{eff}} = \frac{2}{A_i}\left[Z_i Y_q + N_i(1 - Y_q)\right] n_b$
    $\Rightarrow$ asymmetry of medium
  - Δm_i^{Coul} in Wigner-Seitz approximation
  - full coupling of nucleons in clusters to meson fields



Mass Shifts III

TECHNISCHE UNIVERSITÄT DARMSTADT

- light nuclei and NN scattering states
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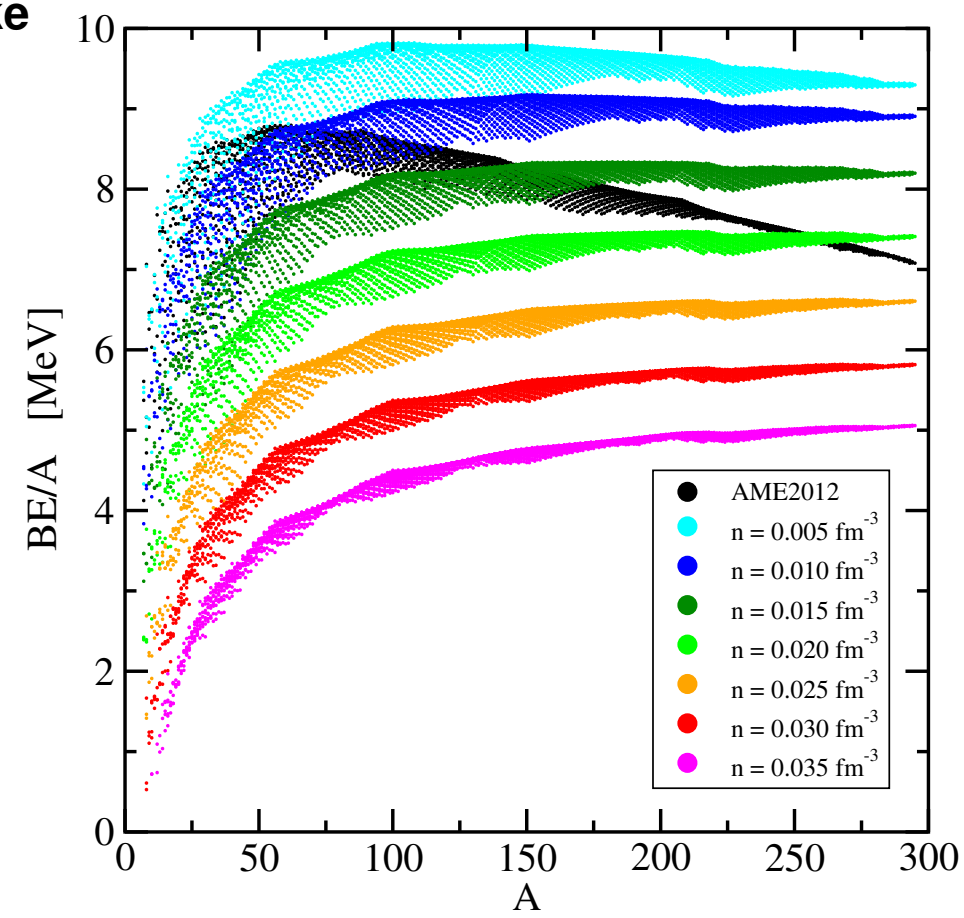
  - Δm_i^{Coul} in Wigner-Seitz approximation
  - full coupling of nucleons in clusters to meson fields
- heavy nuclei
  - heuristic parametrisation



Mass Shifts IV

TECHNISCHE UNIVERSITÄT DARMSTADT

- **nuclear matter**
  - liquid-gas phase transition
  - separation of low- and high-density phases
  - no surface or Coulomb effects

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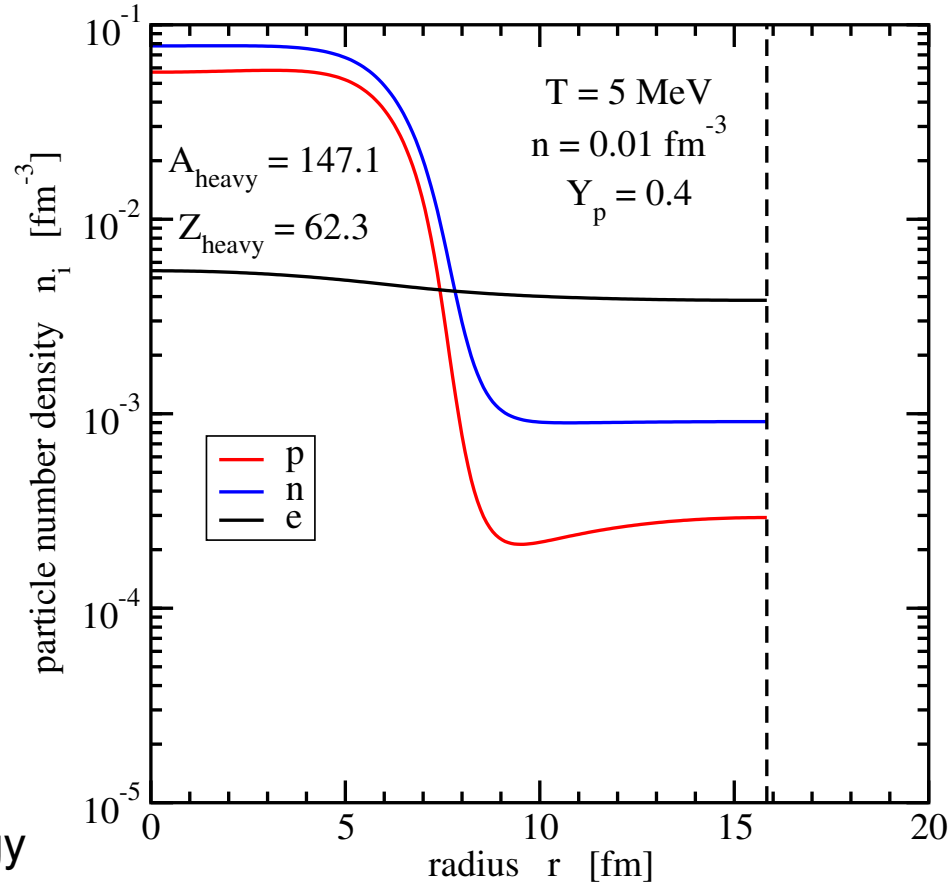
  - effective binding energy from energy difference to homogeneous matter



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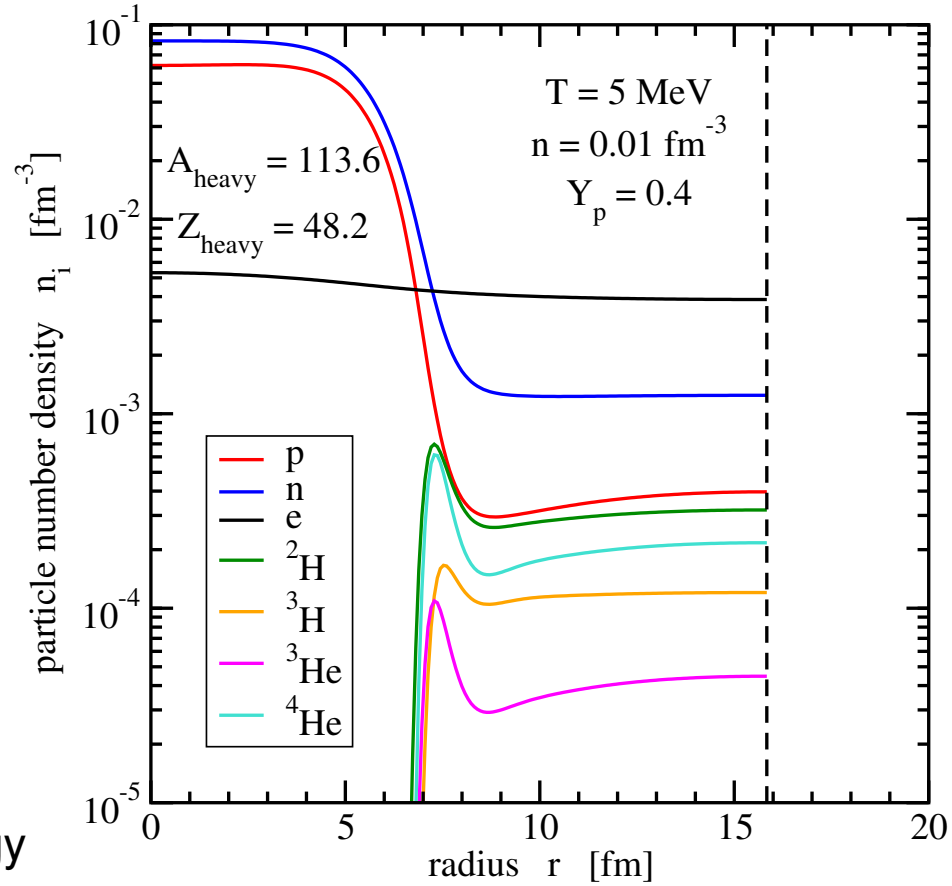

Light Clusters and Continuum Correlations

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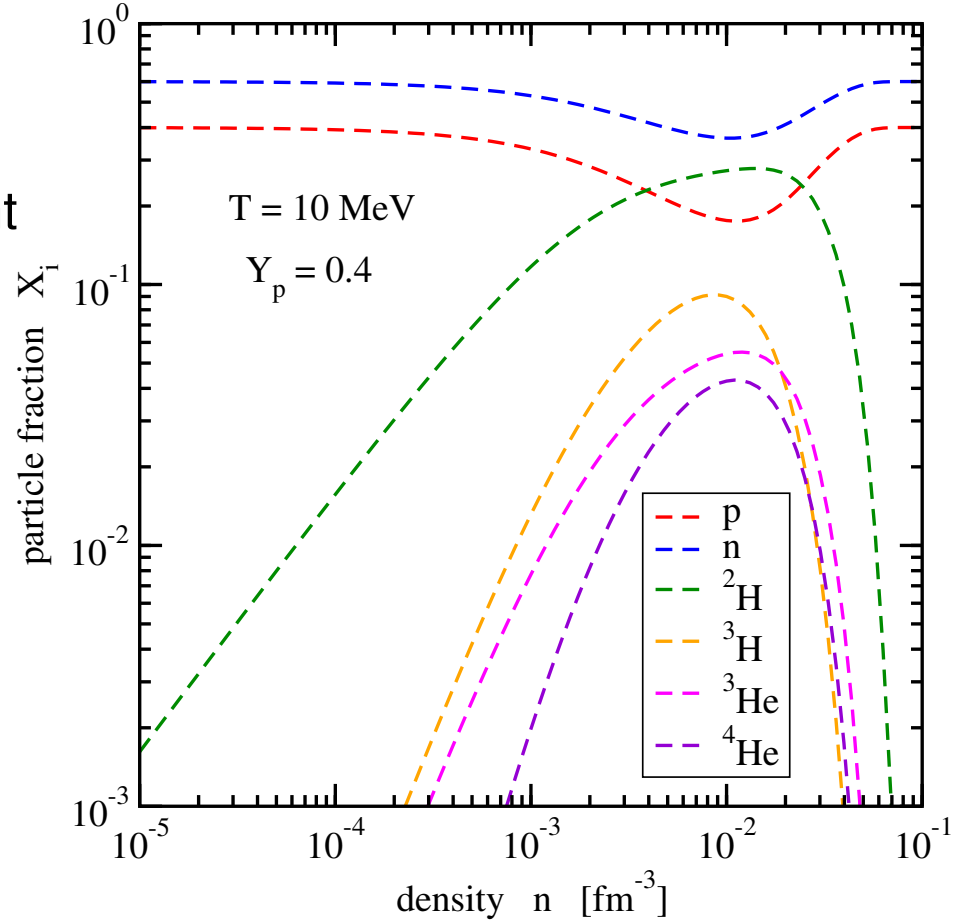
- particle mass fractions
  $X_i = A_i \frac{n_i}{n} \qquad n = n_b = \sum_i A_i n_i$
- low densities:
  two-body correlations most important
- high densities:
  dissolution of clusters
  $\Rightarrow$ Mott effect



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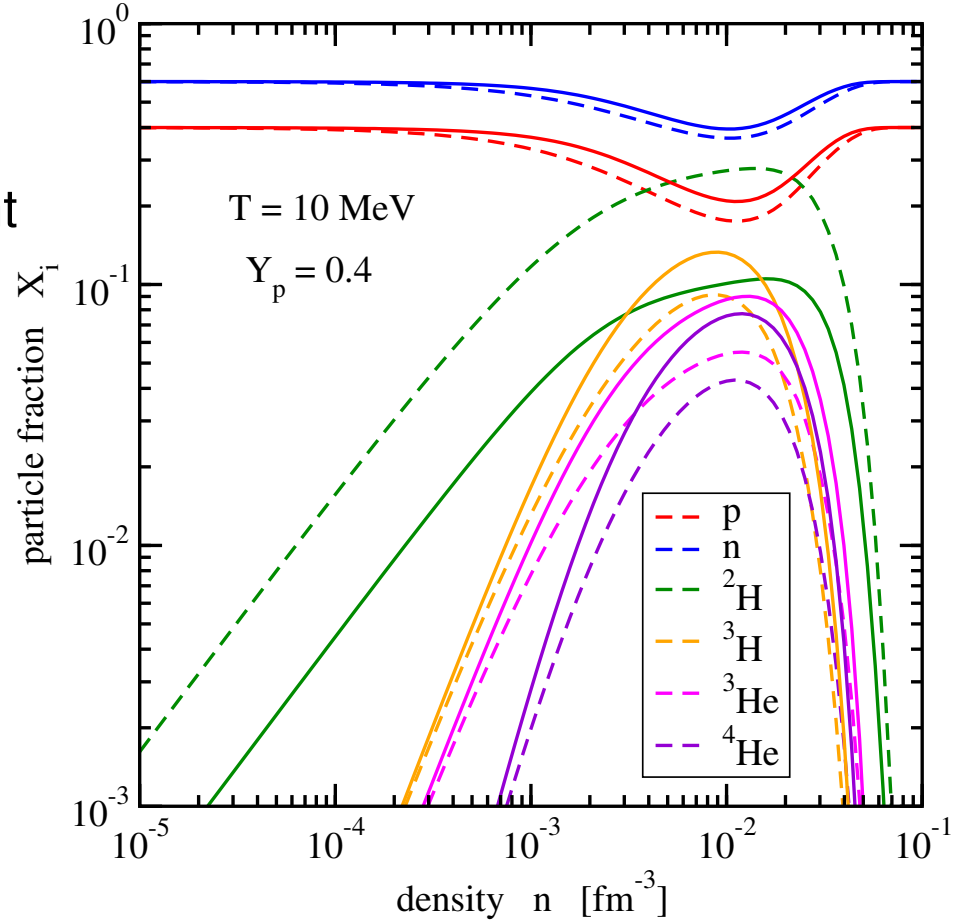
- effect of NN continuum correlations
  - dashed lines: without continuum
  - full lines: with continuum

  $\Rightarrow$ reduction of deutron fraction,
  redistribution of other particles
- correct low-density limit



Light Clusters in Heavy-Ion Collisions

TECHNISCHE UNIVERSITÄT DARMSTADT

emission of light nuclei

- determination of density and temperature of source

  S. Kowalski et al. PRC 75 (2007) 014601

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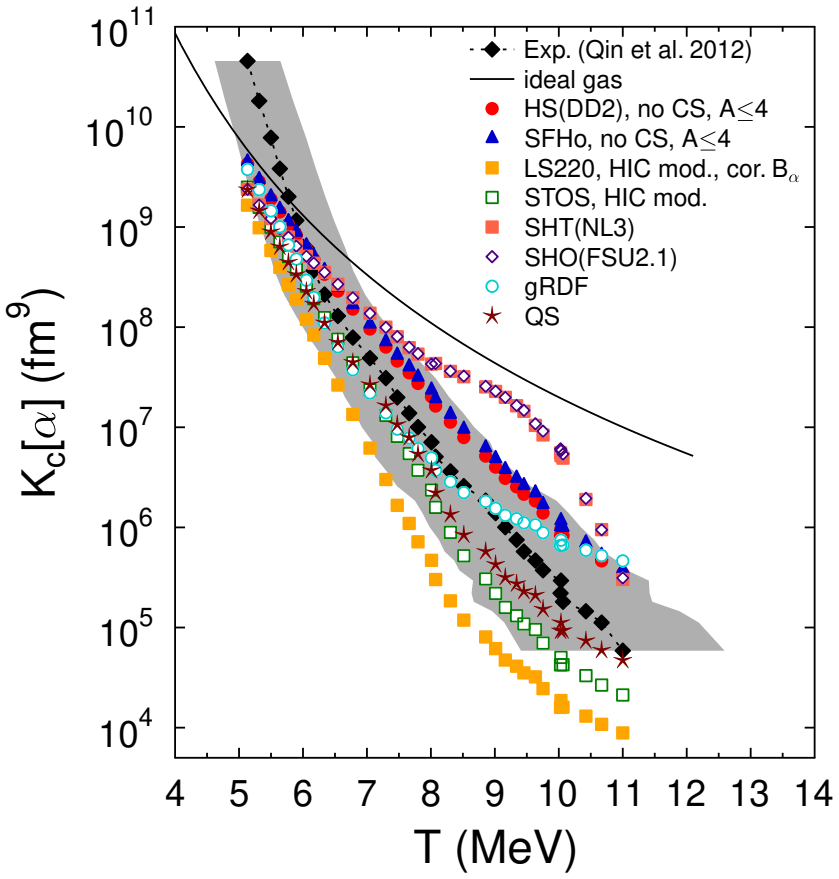
- thermodynamic conditions as in neutrinosphere of core-collapse supernovae
- particle yields $\Rightarrow$ **chemical equilibrium constants** $K_c[i] = n_i/(n_p^{Z_i} n_n^{N_i})$

  L. Qin et al., PRL 108 (2012) 172701

- mixture of ideal gases not sufficient



M. Hempel, K. Hagel, J. Natowitz, G. Röpke, S. Typel, PRC C 91 (2015) 045805

Compact Star Matter

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- number of independent **chemical potentials**
  = number of **conserved charges**
  - baryon number $\rightarrow$ baryon chemical potential μ_B
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- **chemical equilibrium** $\Rightarrow$ relation of chemical potentials
  $\mu_i = B_i\mu_B + Q_i\mu_Q + L_{ei}\mu_{L_e} + L_{\mu i}\mu_{L_\mu} + S_i\mu_S$
  with baryon, charge,... numbers B_i, Q_i, ... of particle i

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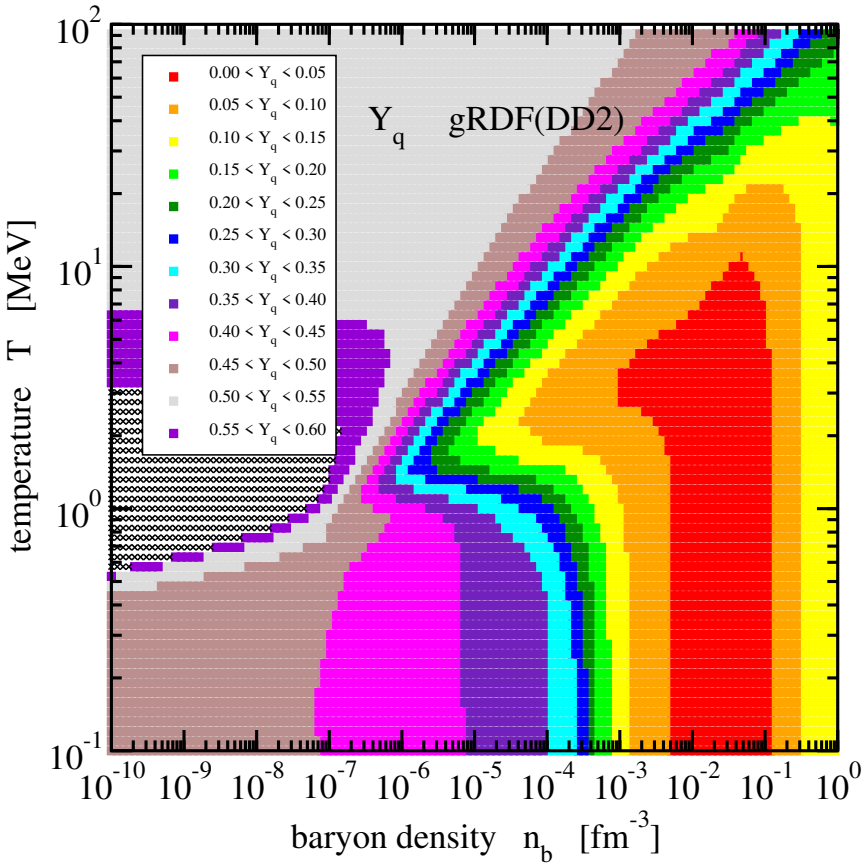
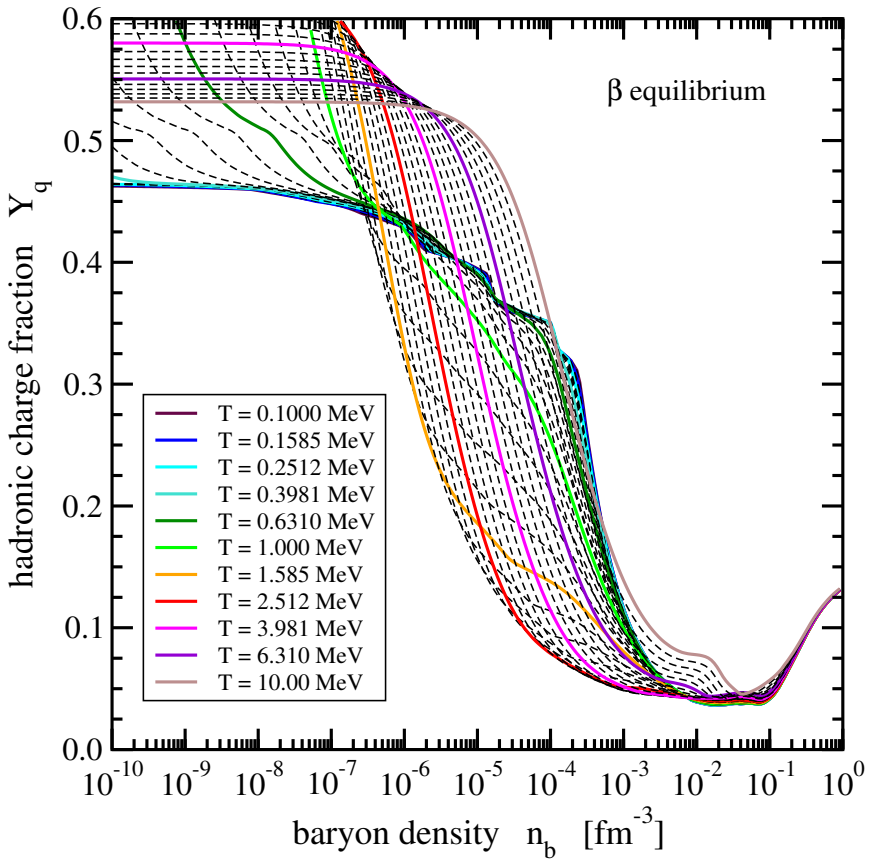
  with baryon, charge,... numbers B_i, Q_i, ... of particle i
- condition of **charge neutrality** fixes μ_Q
- condition of β **equilibrium** (compact stars) fixes $\mu_{L_e} = 0$ (usually $\mu_{L_\mu} = \mu_{L_e}$)
  $\Rightarrow$ only one independent chemical potential (μ_B)

Global EoS for Astrophysical Applications I
Compact Star Matter

TECHNISCHE UNIVERSITÄT DARMSTADT

- **hadronic charge fraction $Y_q = \sum_i Q_i n_i / n_b$** (without leptons)
  $\Rightarrow$ neutronisation with increasing baryon density

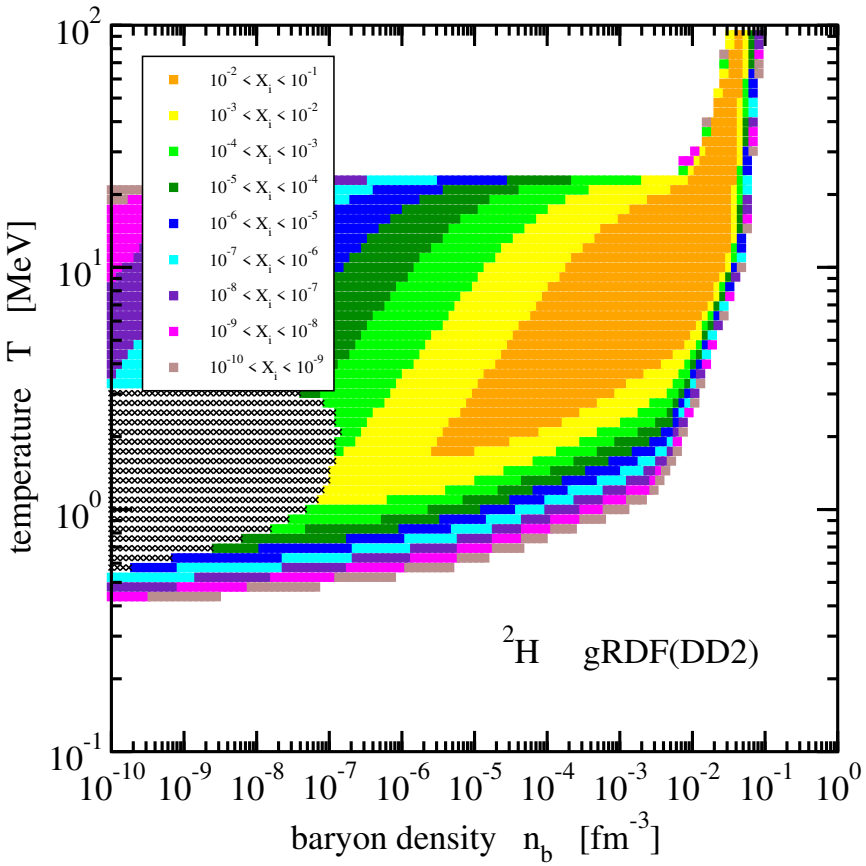


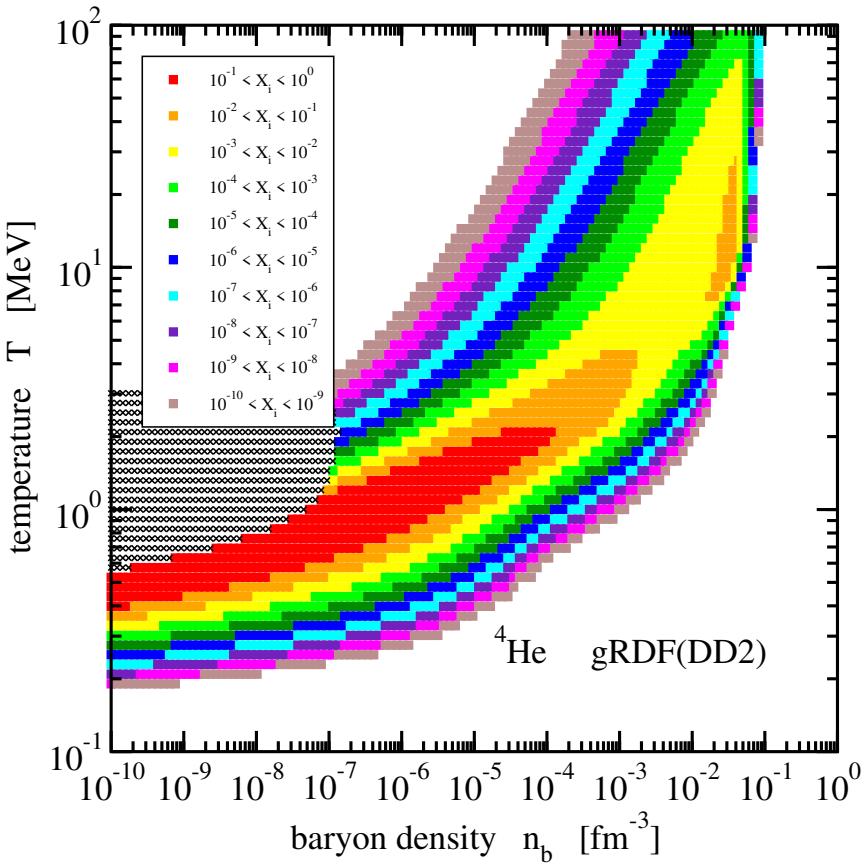


Global EoS for Astrophysical Applications II
Compact Star Matter

TECHNISCHE UNIVERSITÄT DARMSTADT

- **mass fractions $X_i = A_i n_i / n_b$ of ^{2}H and ^{4}He**

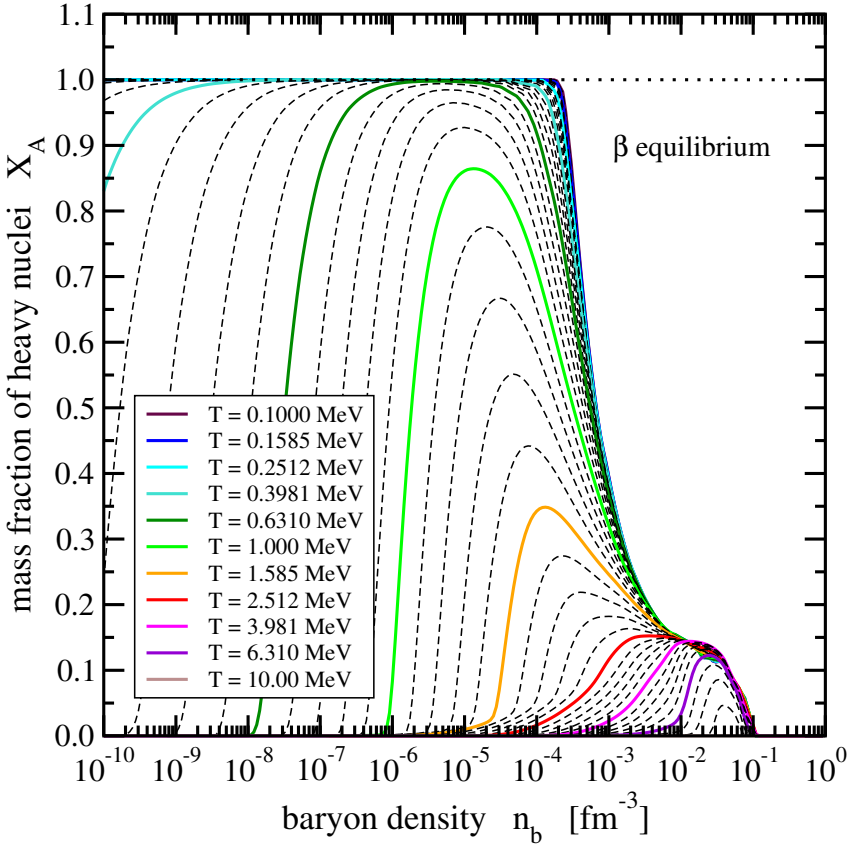


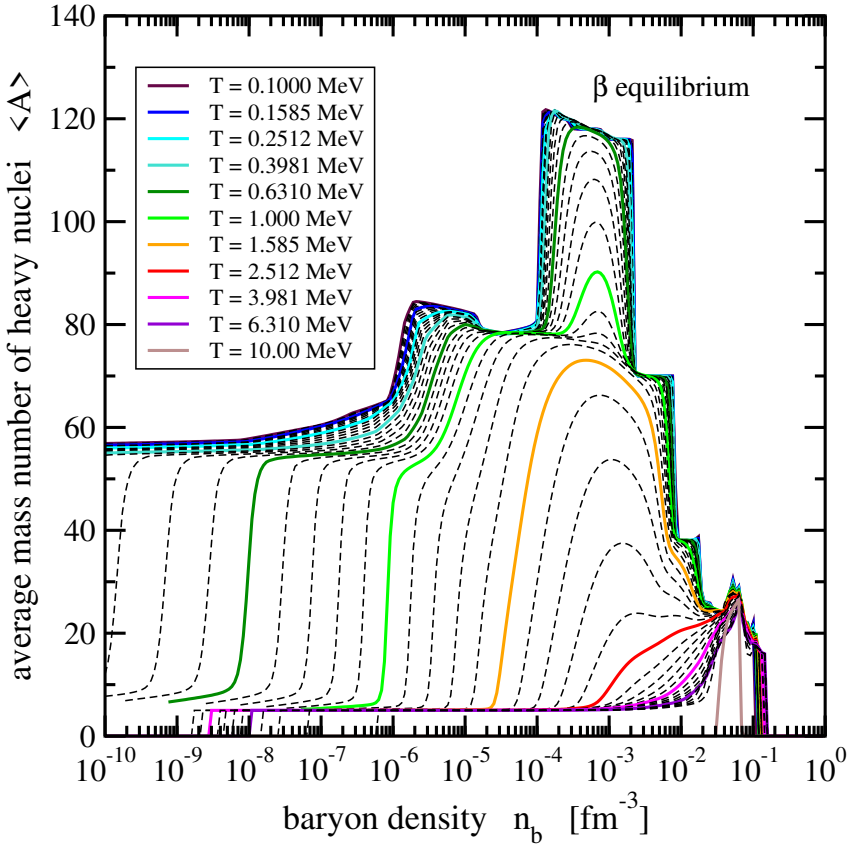


Global EoS for Astrophysical Applications IV
Neutron Star Matter

TECHNISCHE UNIVERSITÄT DARMSTADT

- **mass fraction X_{heavy} and average mass number $\langle A \rangle$ of heavy nuclei**

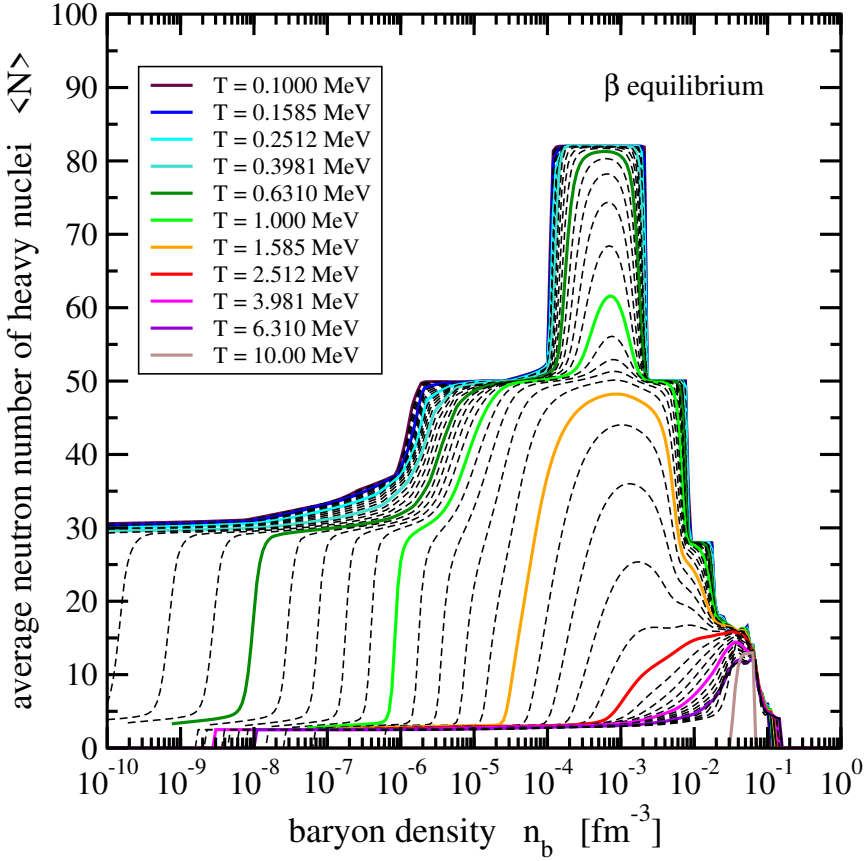


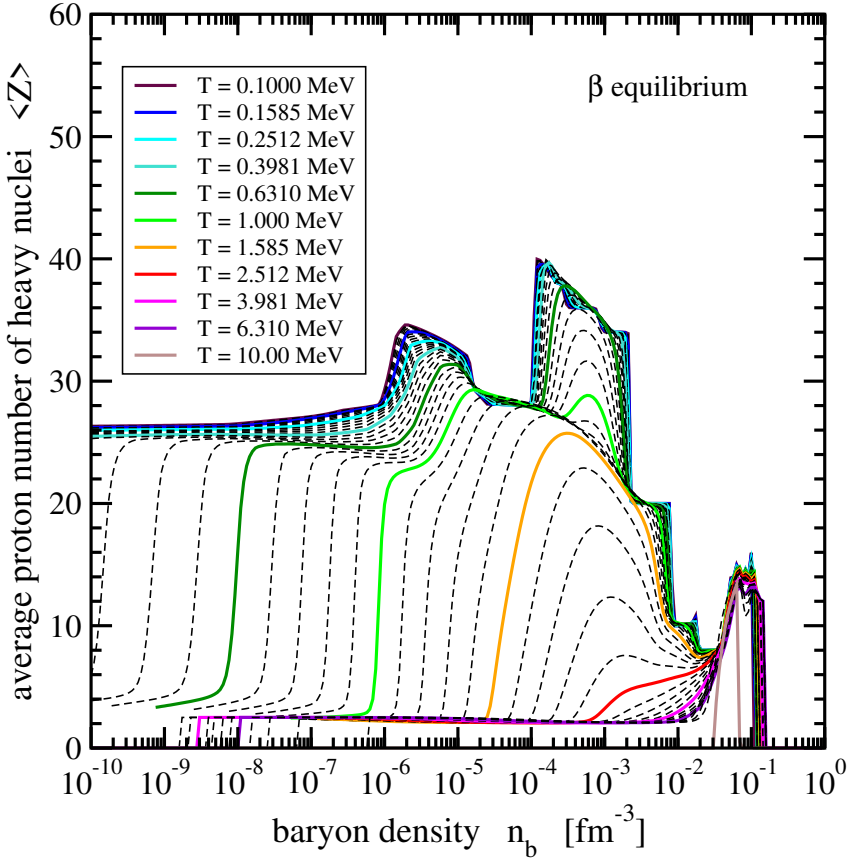


Global EoS for Astrophysical Applications V
Neutron Star Matter

TECHNISCHE UNIVERSITÄT DARMSTADT

- **average neutron ⟨N⟩ and charge number ⟨Z⟩ of heavy nuclei**





Low-Temperature Limit I

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- **phase transition** from gas/liquid phase to solid phase
- **correlations from Coulomb interaction** essential
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- essential quantity: **plasma parameter**
  $\Gamma = Z_{\text{ion}}^{5/3} e^2/(a_e T)$ with $a_e = [3n_e/(4\pi)]^{1/3}$

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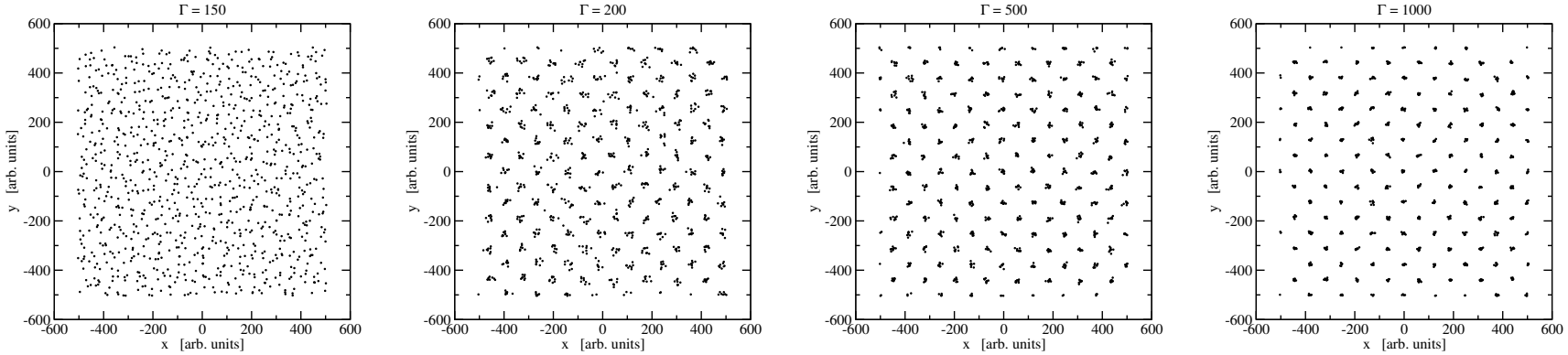
- Monte-Carlo simulations (molecular dynamics)
- example: one-component plasma (OCP), 1024 ions in $8 \times 8 \times 8$ bcc lattice



Low-Temperature Limit II

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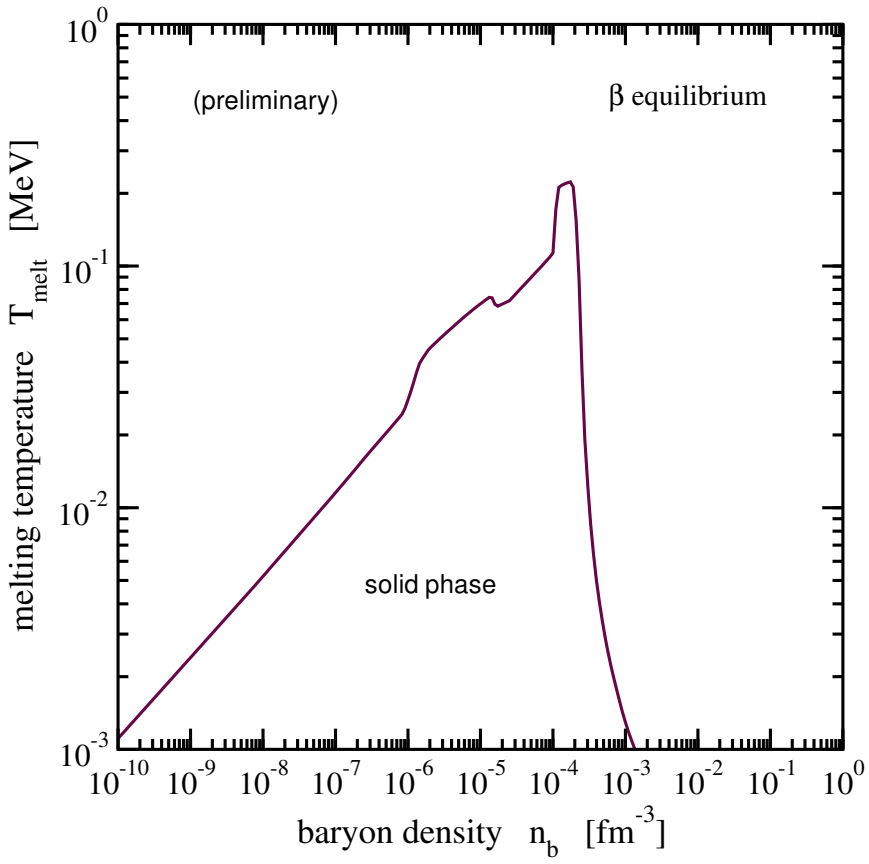
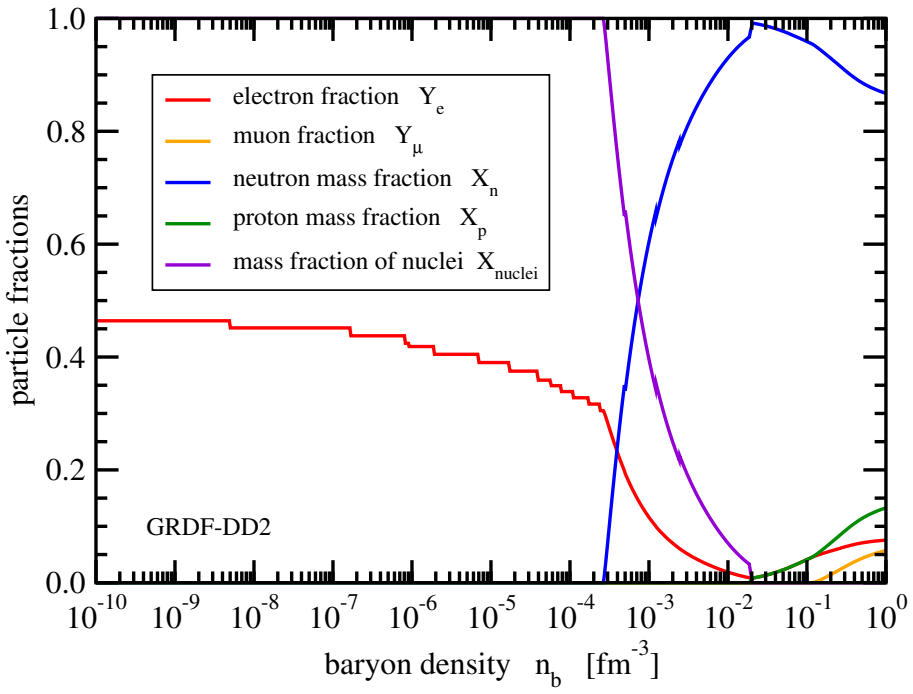
- phase transition from gas/liquid phase to solid phase
- melting point at $\Gamma \approx 175$
- neutron star:
  $T \approx 0 \Rightarrow$ formation of crystal
  (neutron star crust)

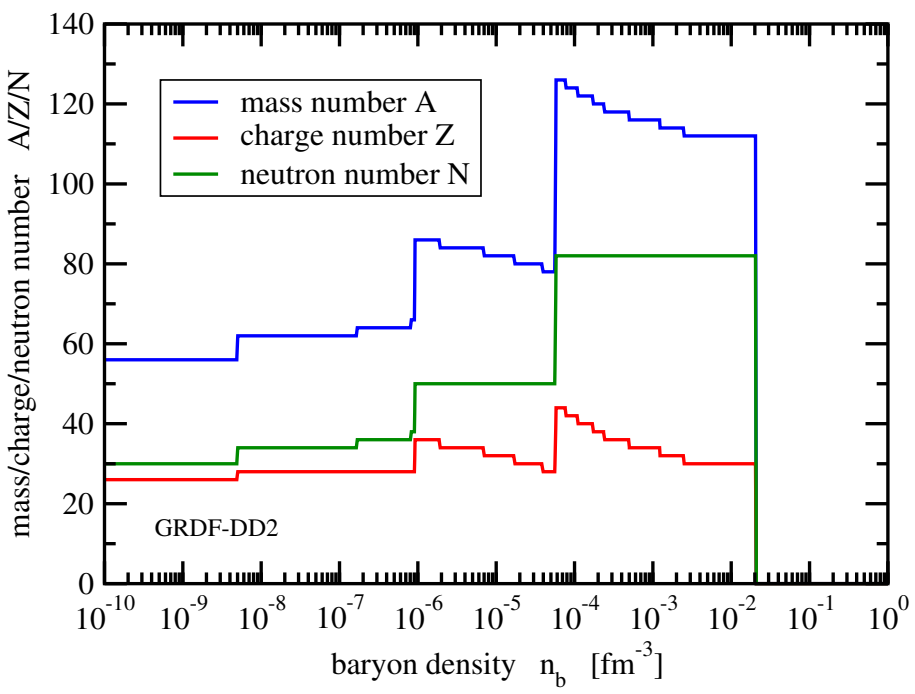


Compact Star Matter Equation of State – Low Densities

TECHNISCHE UNIVERSITÄT DARMSTADT

- temperature $T = 0$, β equilibrium
- sequence of ions in background of electrons, phase transitions
- free neutrons above neutron drip density





Structure of Neutron Stars I

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- mass-radius relation in **general relativity**
  $\Rightarrow$ Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{dP}{dr} = -G\frac{M(r)\varepsilon(r)}{c^2 r^2}\left[1 + \frac{P(r)}{\varepsilon(r)}\right]\left[1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right]\left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1}$$

with energy density $\varepsilon(r)$ and mass $M(r) = \frac{4\pi}{c^2}\int_0^r dr'\,(r')^2\,\varepsilon(r')$ inside radius r

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  with energy density $\varepsilon(r)$ and mass $M(r) = \frac{4\pi}{c^2}\int_0^r dr'\,(r')^2\,\varepsilon(r')$ inside radius r

- solution with **equation of state** $\varepsilon = \varepsilon(\varrho) \Rightarrow P = \varrho^2 \frac{d(\varepsilon/\varrho)}{d\varrho}$,
  initial condition $\varrho(0)$ (central density),
  and integration up to neutron star radius R where $P(R) = 0$
  $\Rightarrow$ sequence of **spherical, non-rotating neutron stars**
  with mass $M = M(R)$ and radius R

Structure of Neutron Stars II

TECHNISCHE UNIVERSITÄT DARMSTADT

- GRDF-DD2 at zero temperature
- unified equation of state
- solution of TOV equation
  $\Rightarrow$ mass-radius relation
  $M_{\mathrm{max}} = 2.42\ \mathrm{M}_{\odot}$, $R_{1.4} = 13.2$ km

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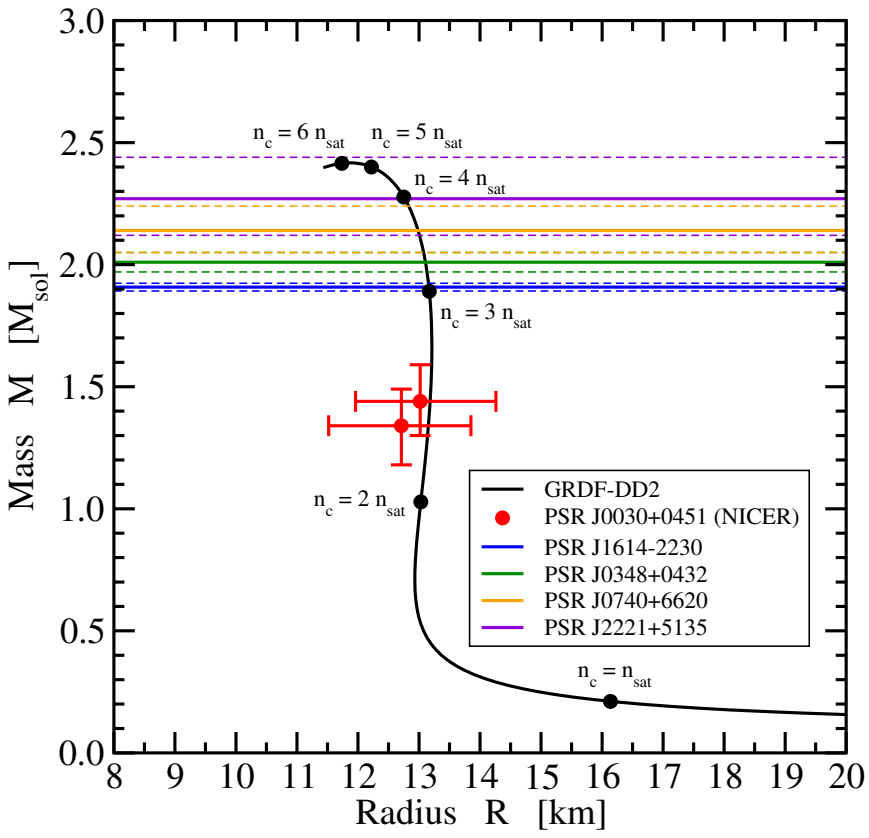
  $M_{\text{max}} = 2.42\ M_\odot$, $R_{1.4} = 13.2$ km
- largest observed masses

$(1.908 \pm 0.016)\ M_\odot$	(PSR J1614-2230)
$(2.01 \pm 0.04)\ M_\odot$	(PSR J0348+0432)
$2.14^{+0.10}_{-0.09}\ M_\odot$	(PSR J0740+6620)
$2.27^{+0.17}_{-0.15}\ M_\odot$	(PSR J2221+5135)

- NICER@ISS:
  mass and radius of PSR J0030+0451
  T. E. Riley et al. ApJL 887 (2019) L21,
  M. C. Miller et al. ApJL 887 (2019) L24



Surface Properties of Heavy Nuclei

Neutron Skins of Heavy Nuclei

TECHNISCHE UNIVERSITÄT DARMSTADT

- neutron-rich nuclei
  $\Rightarrow$ extended density distribution of neutrons
- **neutron skin thickness**
  $S = \Delta r_{np} = r_n - r_p$
  difference of neutron and proton
  root-mean-square radii $r_i = \sqrt{\langle r_i^2 \rangle}$

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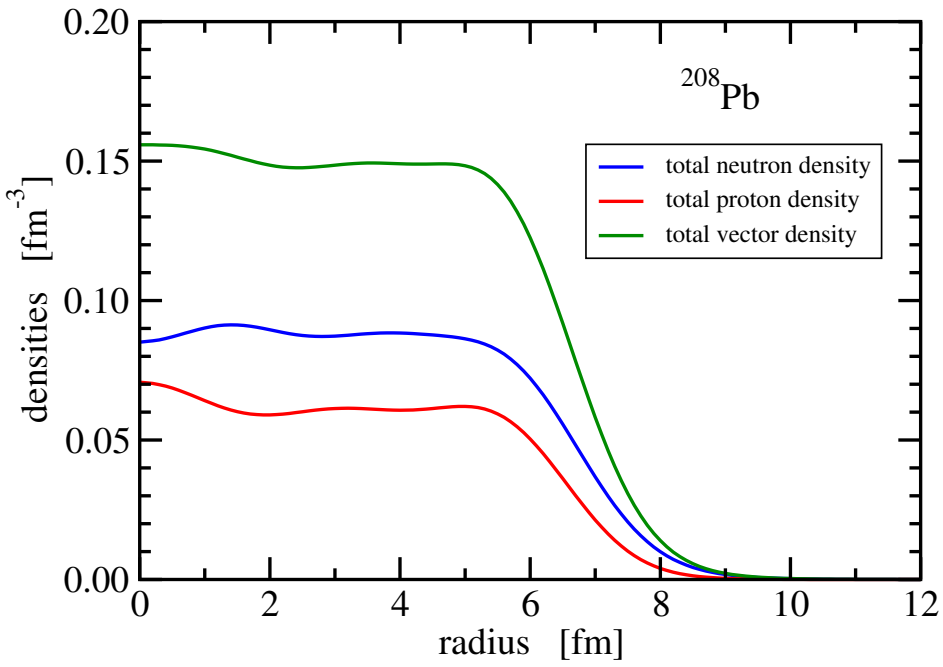
  difference of neutron and proton root-mean-square radii $r_i = \sqrt{\langle r_i^2 \rangle}$
- example:
  ^{208}Pb with DD2 parameters
  - $r_n = 5.682$ fm
  - $r_p = 5.484$ fm

  $\Rightarrow \Delta r_{np} = 0.198$ fm



Neutron Skins and Neutron-Matter Equation of State

TECHNISCHE UNIVERSITÄT DARMSTADT

- study of many nonrelativistic Skyrme-Hartree-Fock models

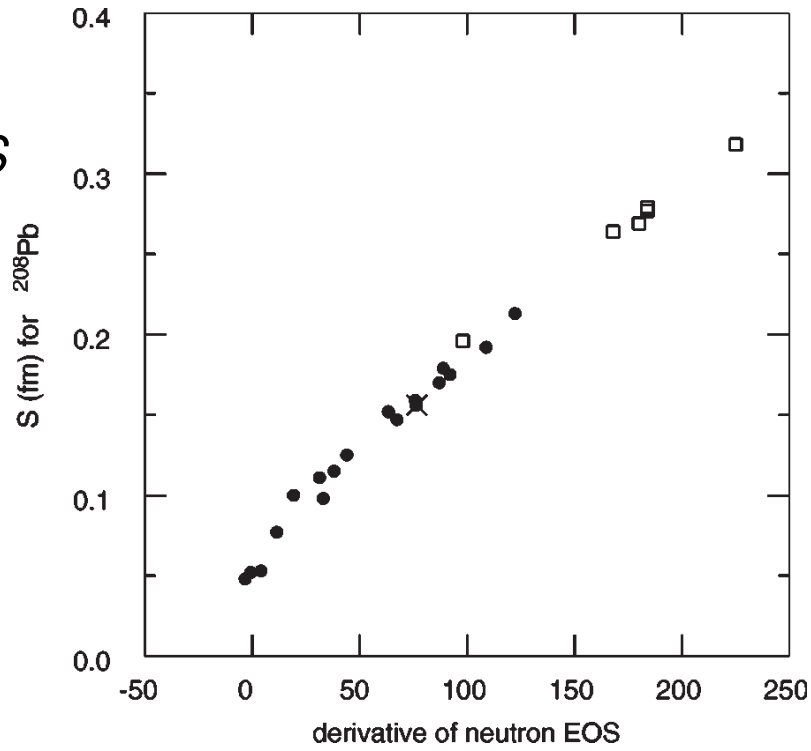
  (B.A. Brown, Phys. Rev. Lett. 85 (2000) 5296)

  ⇒ **correlation** of neutron-skin thickness S with derivative of neutron-matter EoS ≙ pressure of neutron matter

- extension to relativistic mean-field models

  (S. Typel and B.A. Brown, Phys. Rev. C 64 (2001) 027302)

  ⇒ similar trend



Neutron Skin and Symmetry Energy

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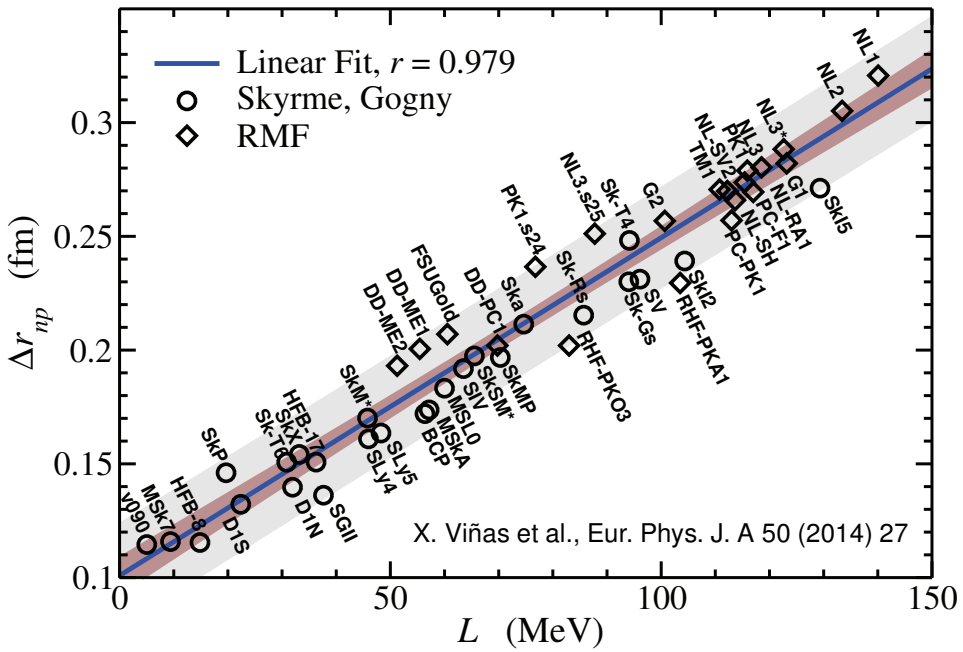
- **correlations**
  - neutron-skin thickness Δr_{np}
    $\leftrightarrow$ pressure P_0 of pure neutron matter at saturation
    $\leftrightarrow$ slope parameter L of symmetry energy
  - confirmed by many models

neutron skin thickness in ^{208}Pb



Neutron Skin and Symmetry Energy

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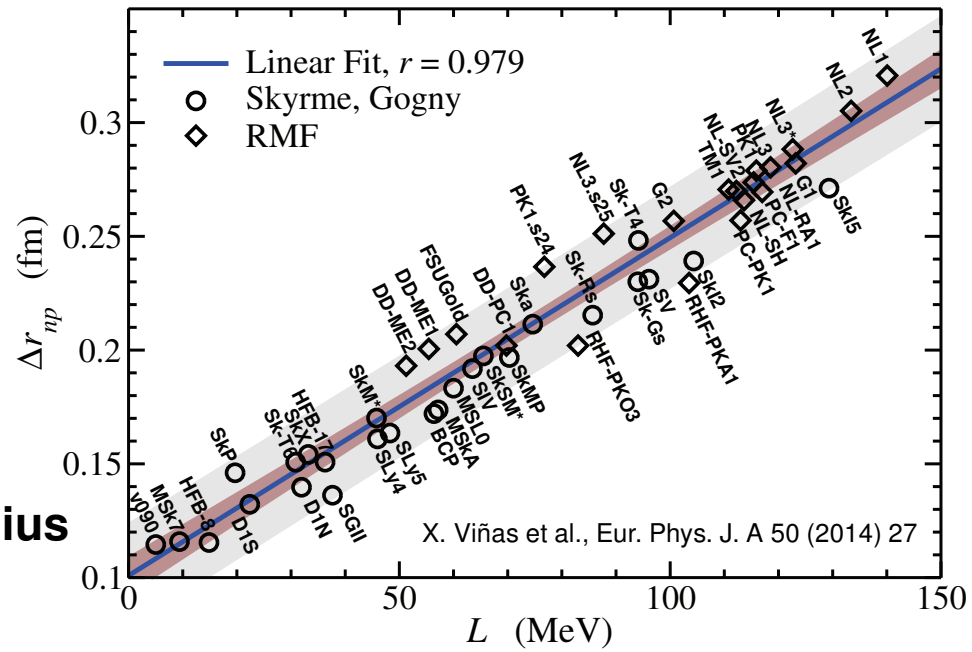
- **correlations**
  - neutron-skin thickness Δr_{np}
    $\leftrightarrow$ pressure P_0 of pure neutron matter at saturation
    $\leftrightarrow$ slope parameter L of symmetry energy
  - confirmed by many models
- experimental determination of neutron skin thickness Δr_{np}:
  - measurement of **neutron rms radius** e.g. parity violation in electron scattering on ^{208}Pb (PREX)
    (D. Adhikari et al., Phys. Rev. Lett. 126 (2021) 172502)
  - many other indirect methods
- effects of **correlations?**

neutron skin thickness in ^{208}Pb



Application of Generalized Relativistic Density Functional (GRDF)

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- used degrees of freedom:
  neutrons, protons, α particles
- extended Thomas-Fermi approximation for nucleons (fermions)
- explicit wavefunction of α particle (boson) in WKB approximation

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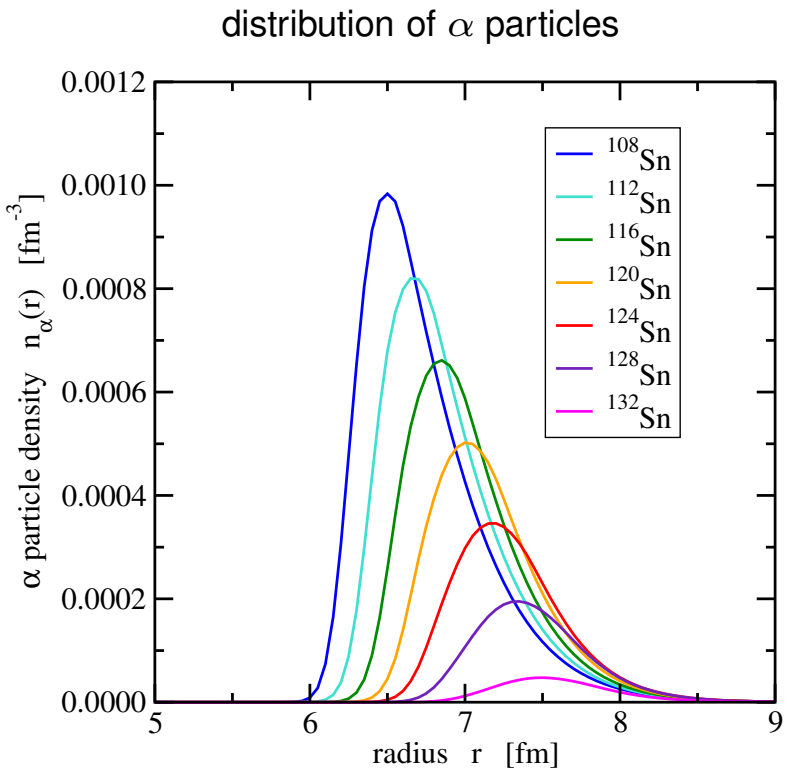
- study of chain of **Sn nuclei**

  (S. Typel, Phys. Rev. C 89 (2014) 064321)

  $\Rightarrow$ suppression of α particles at high nucleon densities

  $\Rightarrow$ α particle formation at nuclear surface

  $\Rightarrow$ reduction of α probability with increasing neutron excess



α-Particle Correlations at Surface of Sn Nuclei

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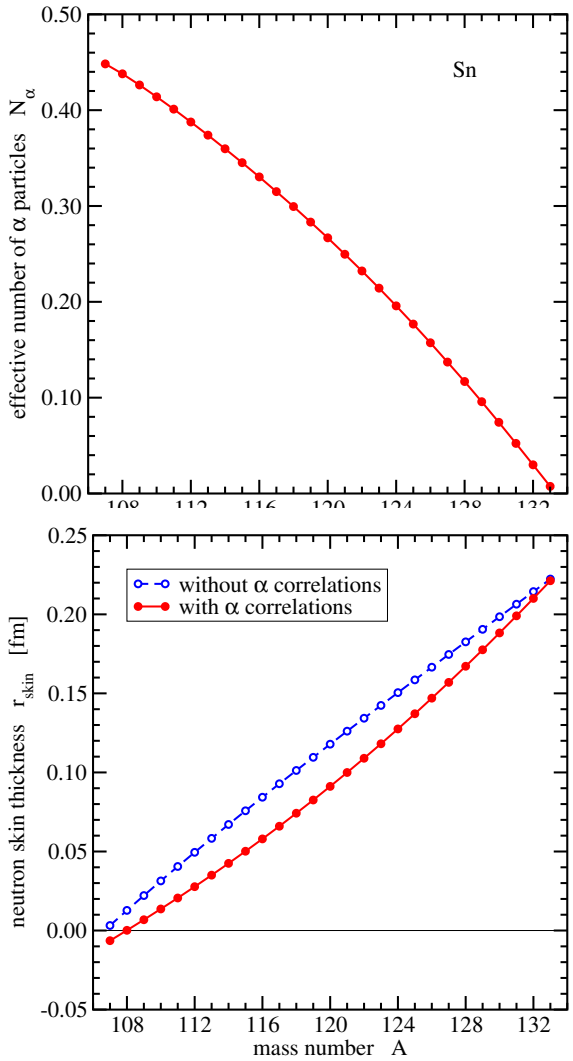
- **prediction** of GRDF
  - decrease of effective number of α particles N_α with neutron excess
  - nuclear surface less isospin asymmetric (α particle is np symmetric)
  - reduction of neutron skin thickness Δr_{np}

$\Rightarrow$ change of $\Delta r_{np} - L$ correlation of mean-field models

(L: slope parameter of symmetry energy)



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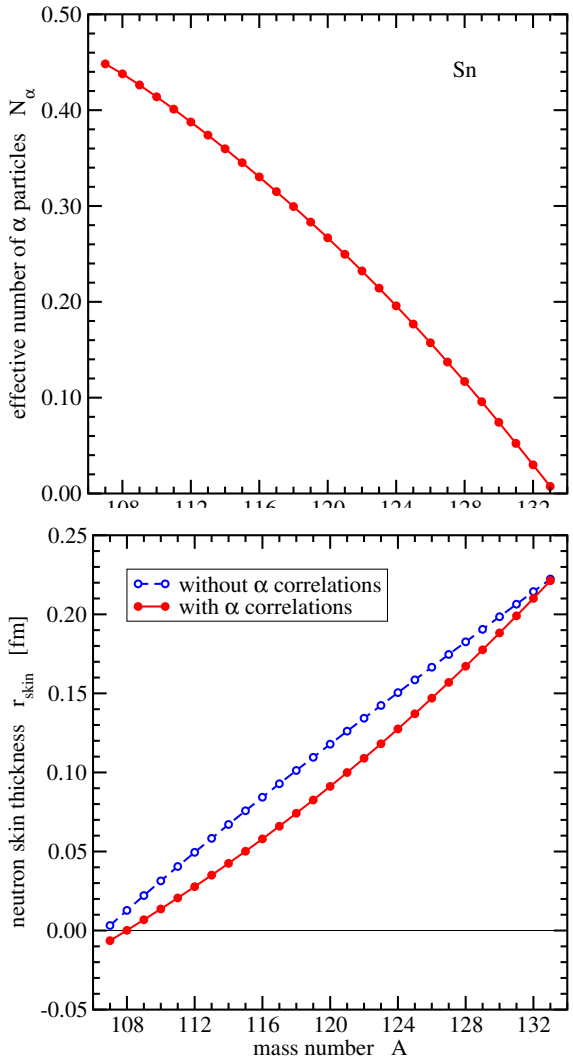
  (L: slope parameter of symmetry energy)
- **experimental test**
  - detect α particles in $(p, p\alpha)$ knockout reactions at quasifree kinematic conditions with chain of Sn isotopes

    $\Rightarrow$ reduction of cross $\sigma \propto N_\alpha$ expected



Study of Correlations at Nuclear Surface I

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- **quasifree (p,pα) knockout reactions on Sn nuclei**
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    - dependence of cross sections on neutron excess
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    - dependence of cross sections on neutron excess
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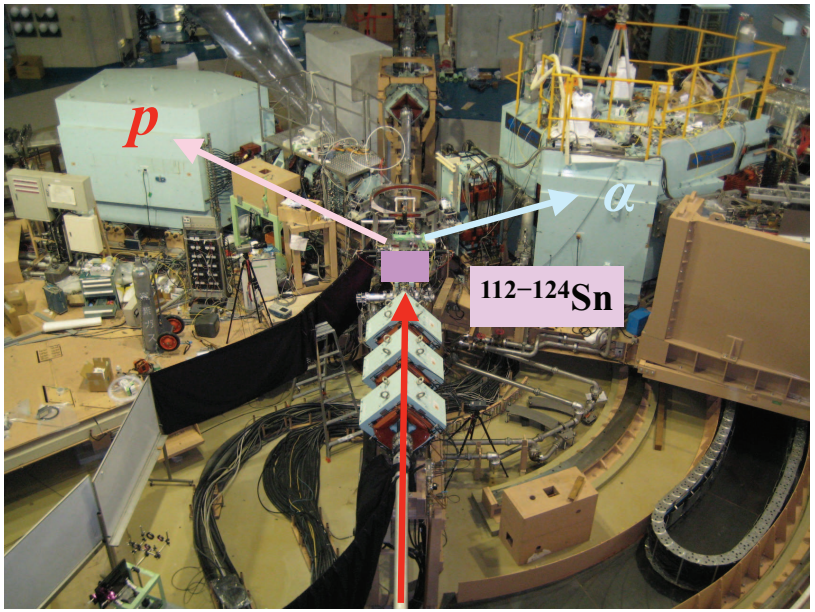
- **experiments at RCNP, Osaka (E461)**
  - targets: stable $^{112-124}$Sn nuclei
  - beam: 392 MeV protons, 100 pnA
  - proton detection: Grand Raiden
  - α detection: LAS
  - first experiment (June 2015): failure of some detectors
  - second experiment (February 2018): successful (Y. Tanaka et al., Science 371 (2021) 260)



Study of Correlations at Nuclear Surface II

TECHNISCHE UNIVERSITÄT DARMSTADT

- **quasifree (p,pα) knockout reactions on Sn nuclei**
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    - spectrometer setting: $\theta_{\text{lab}}(p)$ = 45.3 deg, $\theta_{\text{lab}}(\alpha)$ = 60 deg
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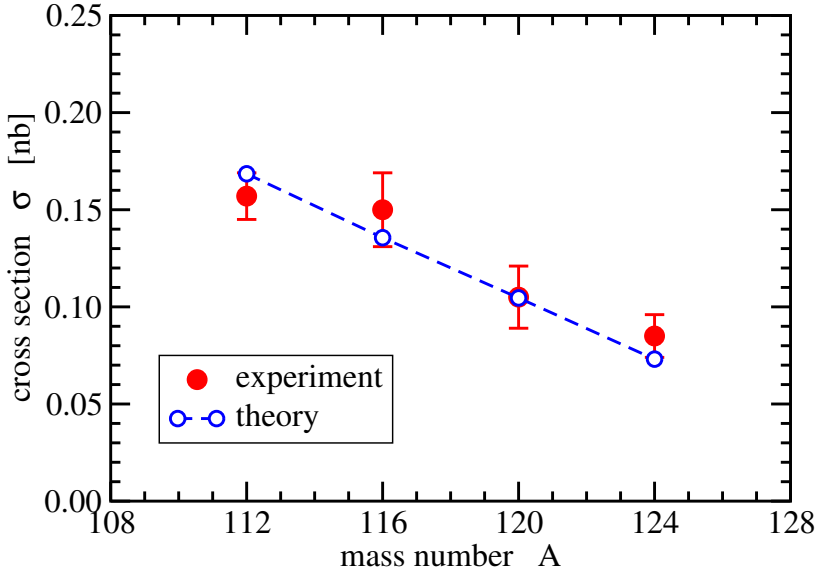
  - theory
    - distorted-wave eikonal model in impulse approximation $\Rightarrow$ factorization of cross section
    - α particle distribution from gRDF
    - proton optical potential from global Dirac phenomenology (S. Hama et al., Phys. Rev. C 41 (1990) 2737)
    - elastic proton-α cross section (K. Yoshida et al., Phys. Rev. C 94 (2016) 044604)
    - scaled α particle optical potential (M. Nolte et al., Phys. Rev. C 36 (1987) 1312)
    - correction for experimental acceptances



Correlations above Nuclear Saturation Density

Correlations at High Densities

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- baryon density n above n_{sat}
  $\Rightarrow$ no clusters as degrees of freedom
  $\Rightarrow$ only single baryons (nucleons, hyperons, ...)
- **microscopic models** (e.g. Brueckner HF)
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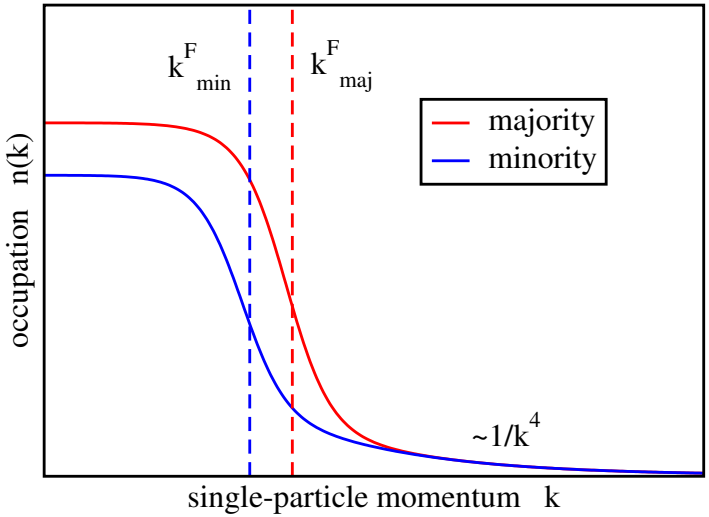

experiments: nucleon knockout from nuclei in inelastic electron scattering
(O. Hen et al. (CLAS Collaboration), Science 346 (2014) 614)
$\Rightarrow$ no sharp cut-off, high-momentum tail

Correlations and Mass Shifts I

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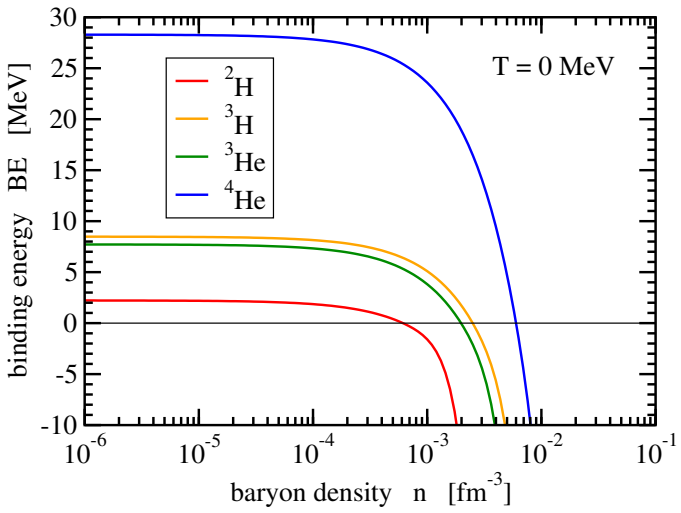
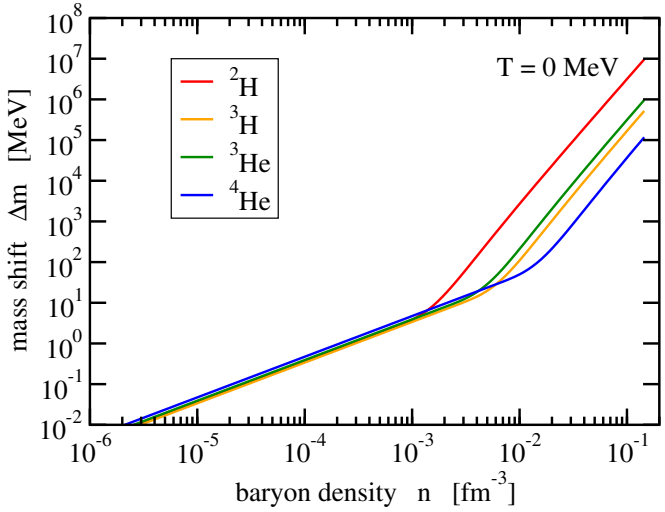
- **choice of density dependence of cluster mass shifts**
  - low densities: linear in n as given by parametrisation of Gerd Röpke
  - higher densities (above Mott density): steeper function ($\propto n^3$, artificial) to avoid reappearance of clusters

$\Rightarrow$ no clusters above saturation density by construction

$\Rightarrow$ transition to mixture of nucleons as quasiparticles





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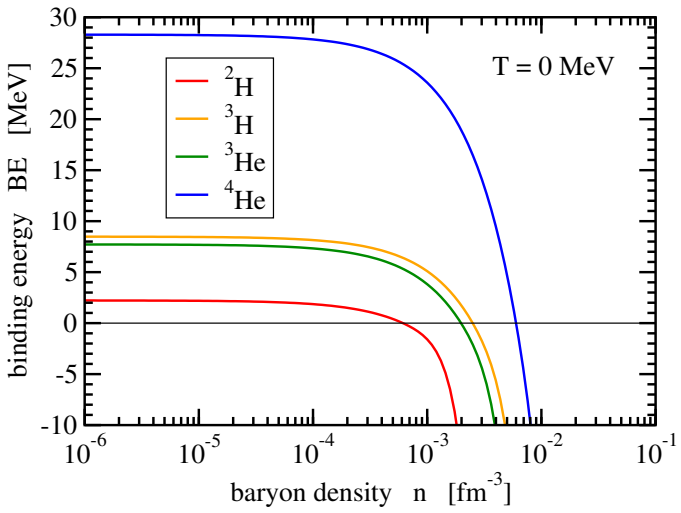
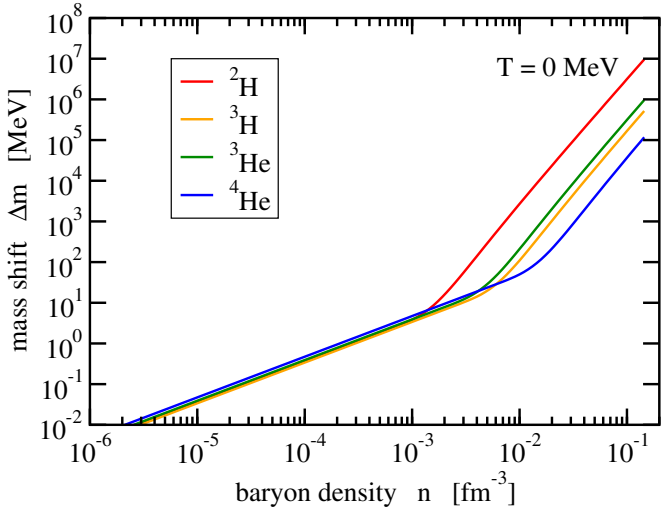
- **representation of many-body correlations above saturation density ?**





Correlations and Mass Shifts II

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- **clusters as effective many-body correlations**
  - internal motion of nucleons in cluster
    $\Rightarrow$ tail in single-nucleon momentum distributions
- **finite temperatures**
  - consider dependence of mass shifts Δm_i
    on cluster cm momentum p_{cm}
    $\Rightarrow$ smaller Δm_i for larger p_{cm}
    $\Rightarrow$ finite cluster density even above n_{sat}

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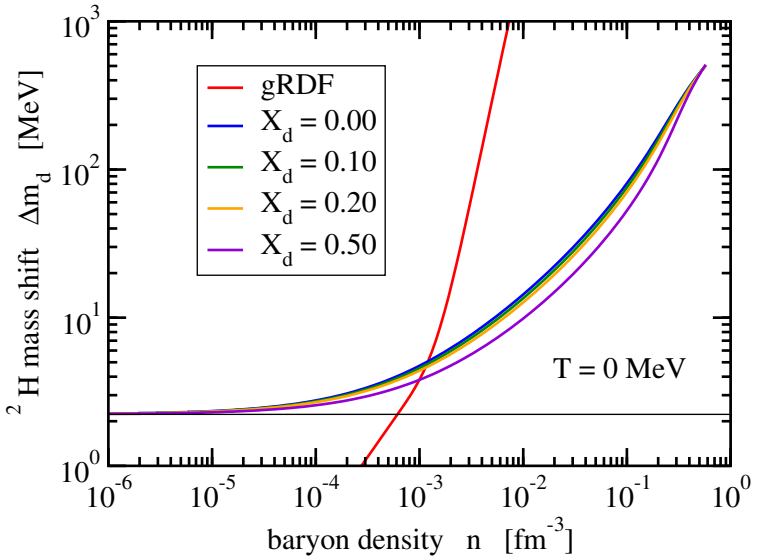
- **zero temperature**
  - no contribution from finite p_{cm}
  - condensation of bosonic clusters
    $\Rightarrow$ condition on chemical potentials μ_i
    $\Rightarrow$ density dependence of Δm_i for given cluster mass fraction $X_i = A_i n_i / n$



$\Rightarrow$ revision of functional form of cluster mass shifts

Conclusions

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- **correlations in strongly interacting matter**
  - essential for thermodynamic properties and composition
  - different types (strong, electromagnetic)
  - theory
    - formation of new degrees of freedom
      $\Rightarrow$ clusters as many-body correlations
    - description of cluster dissolution
      $\Rightarrow$ excluded-volume vs. mass-shift mechanism
  - application
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- **outlook**
  - further extension of relativistic density functional
  - improvement of model parameters
  - change of functional form and density dependence of couplings
  - effective description of correlations above saturation density
  - ...

Related Projects

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- **Collaborative Research Center (CRC) 1245**
  "Nuclei: From Fundamental Interactions to Structure and Stars"
  - Institute for Nuclear Physics, Technical University of Darmstadt
  - funded by Deutsche Forschungsgemeinschaft/German Research Foundation (DFG)
  - `www.sfb1245.tu-darmstadt.de`
- **Research Cluster ELEMENTS**
  "Exploring the Universe from Microscopic to Macroscopic Scales"
  - collaboration of Goethe University Frankfurt, Technical University of Darmstadt, Justus-Liebig University Gießen, GSI Helmholtz Centre for Heavy-Ion Research
  - funded by State of Hesse
  - supported by Helmholtz Research Academy Hesse for FAIR (HFHF)
  - `elements.science`

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Thank You for Your Attention!