How does subatomic matter organize itself?

A low-energy nuclear physics perspective

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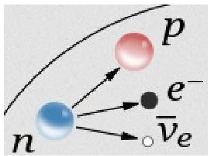
Where can we find neutrons and protons? And in which form? Free? In clusters?

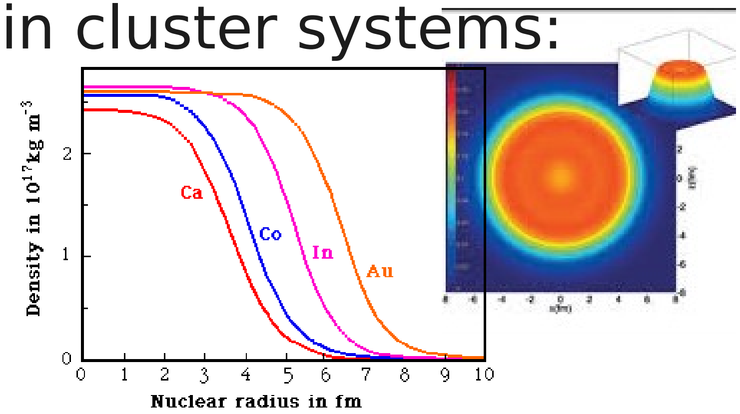
- Neutrons and protons in **Earth** are found in cluster systems: **<u>nuclei</u>**



→ The **interior** of all nuclei has **constant density** (10^{14} times denser than water) named **saturation density**

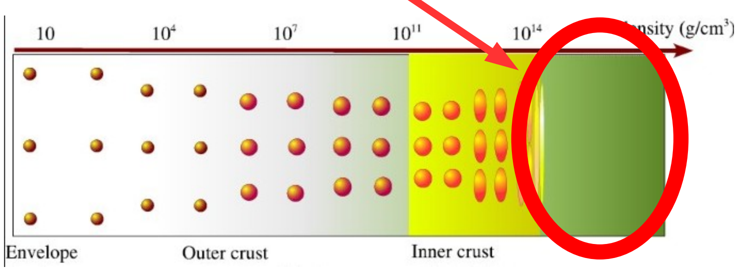
→ Saturation is originated from the **short range** nature of the **nuclear effective interaction**

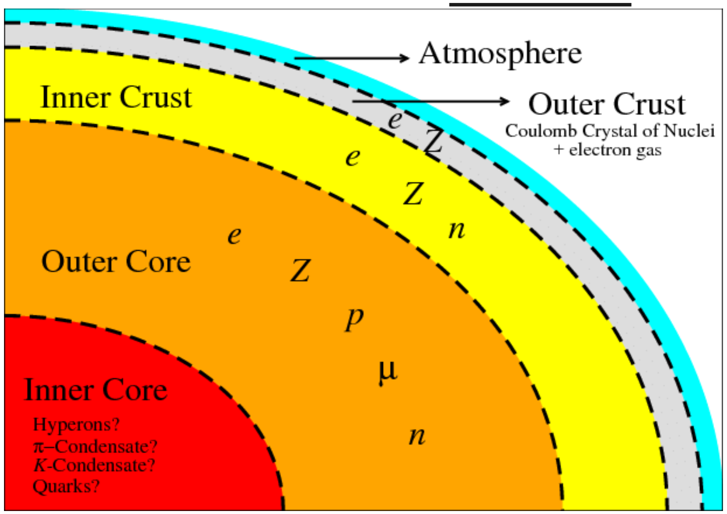
→ Neutron in 15 minutes must find a proton or …



- In **heavens**, neutrons and protons can be also found as an interacting and unbound Fermi liquid: **matter in the <u>outer core</u> of a neutron star**

→ Densities can reach several times nuclear saturation

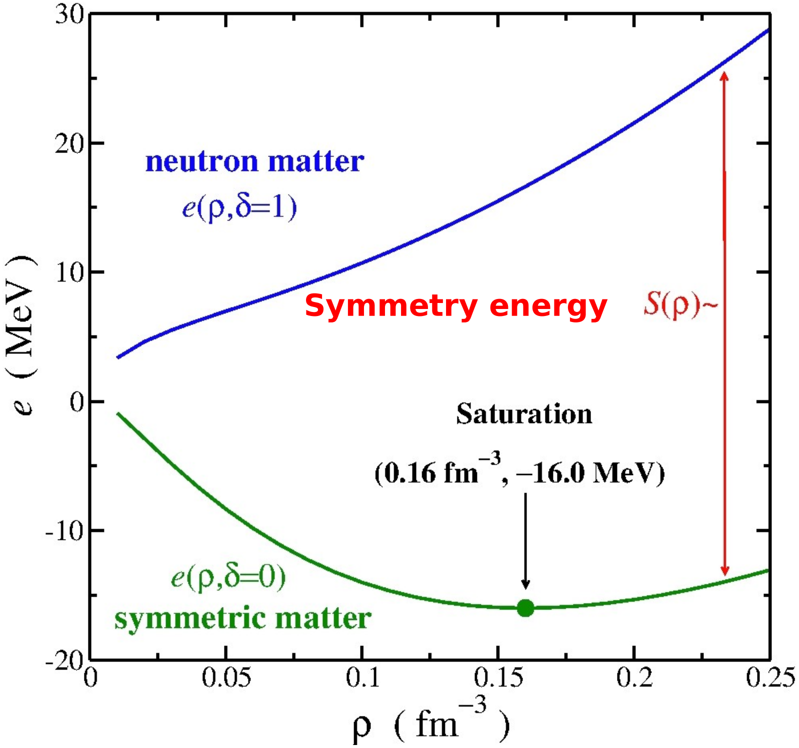




Nuclear Equation of State (EoS)

Unpolarized **nuclear matter** at zero temperature (10^{10}K→1MeV) is defined as the **energy** per **nucleon** (*e*) as a function of the **neutron** (ρ_n) and **proton** (ρ_p) **densities** as (*isospin conserving* $V_{nn} = V_{pp} = V_{np}$):

$$\boxed{e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2} + \mathcal{O}[\delta^4] \quad \text{where } \rho = \rho_n + \rho_p \text{ and } \delta = \frac{\rho_n - \rho_p}{\rho}$$



It is customary to **expand *e(ρ,δ)*** around nuclear **saturation density** ρ_0 ~ 0.16 fm^{-3}

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho - \rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}[\rho^3, \delta^4]$$

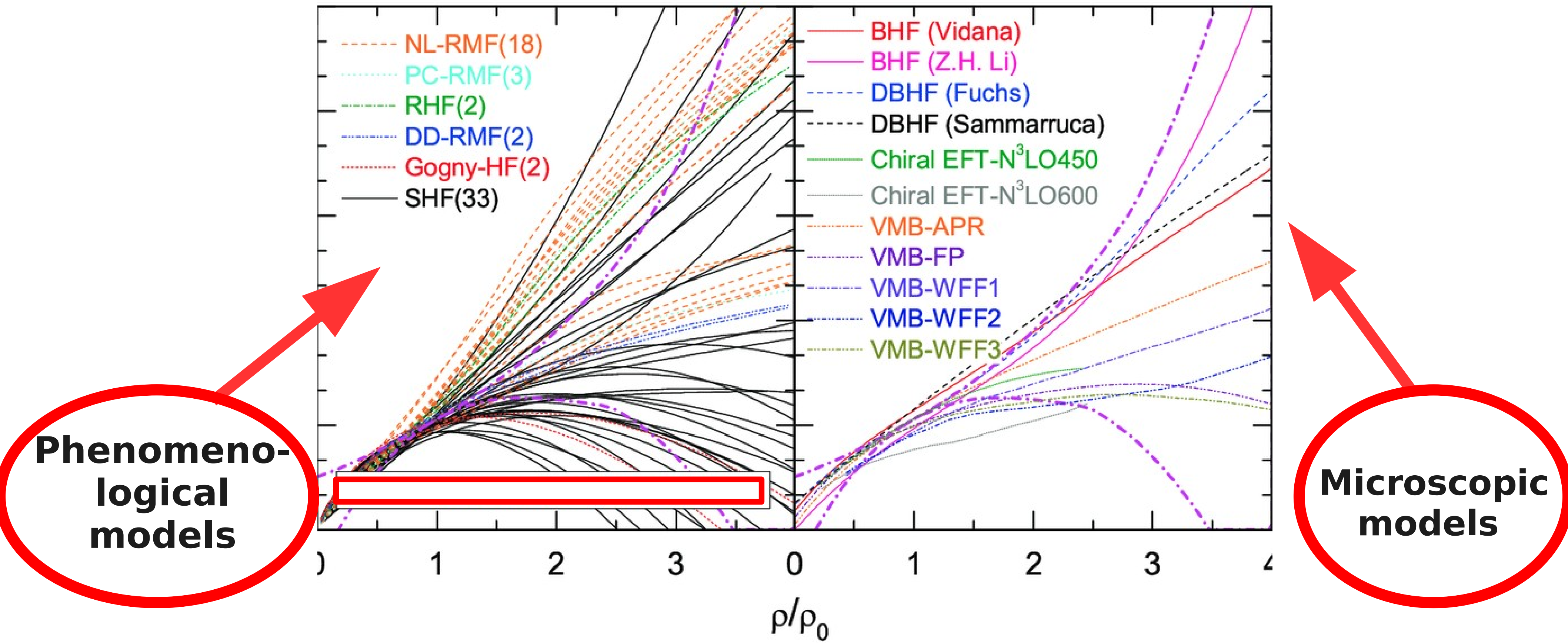
K_0 → how **compressible** is symmetric matter at ρ_0

J → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L → **neutron pressure** in neutron matter at ρ_0

Symmetry energy $S(\rho) \sim e(\rho,\delta=1)-e(\rho,\delta=0)$

Symmetry energy not well constrained



Towards understanding astrophysical effects of nuclear symmetry energy
Bao-An Li, Plamen G. Krastev, De-Hua Wen & Nai-Bo Zhang EPJ A 55, 117 (2019)

What can we learn from the Earth and the Heavens about the Nuclear Equation of State and, thus, how subatomic matter organize itself?

(some examples)

From Heaven: Neutron Star Mass

Nuclear models that account for different nuclear properties on **Earth** predict a large **variety** of **Neutron Star Mass-Radius** relations → **Observation of a 2M_{sun} has constrained nuclear models.**

Tolman–Oppenheimer–Volkoff equation (sph. sym.):

$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r);$$
$$\frac{dP}{dr} = -G\frac{\mathcal{E}(r)M(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$\mathcal{E}(r) \rightarrow$ degeneracy pressure from neutrons $\rightarrow M_{\max} = 0.7 M_{\text{sun}}$

Nuclear Physics input is fundamental

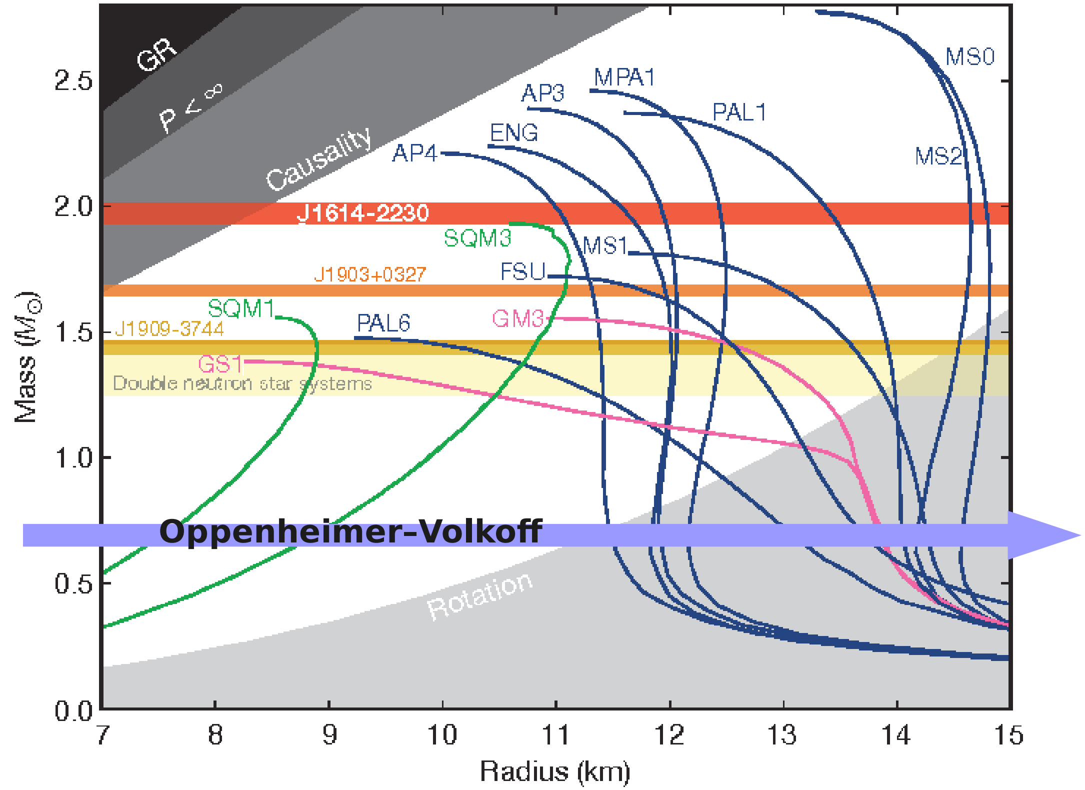
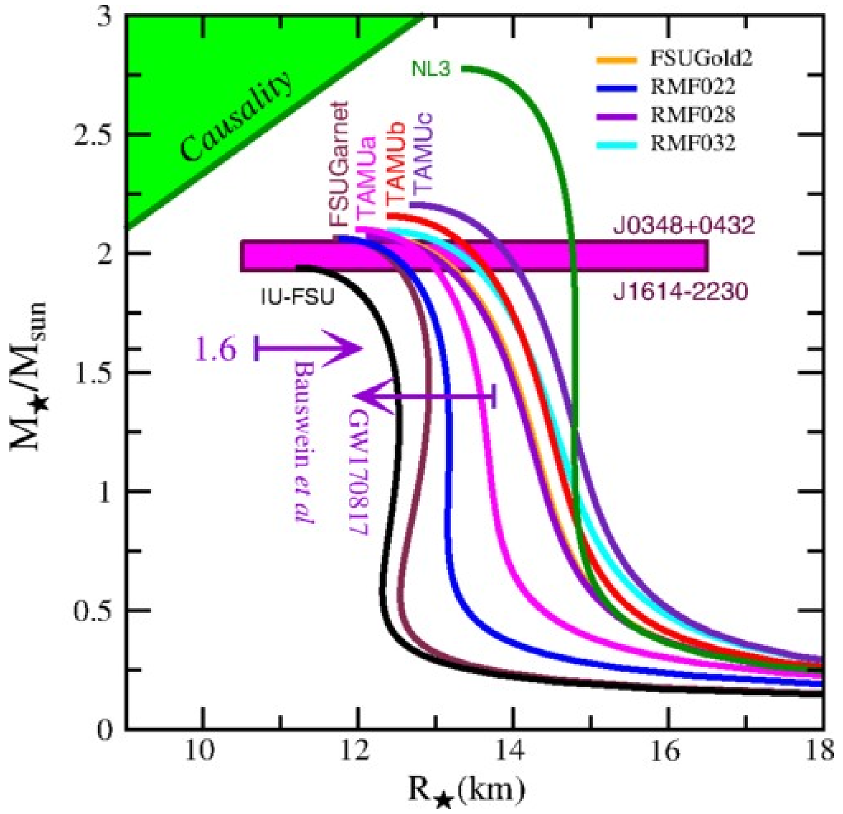


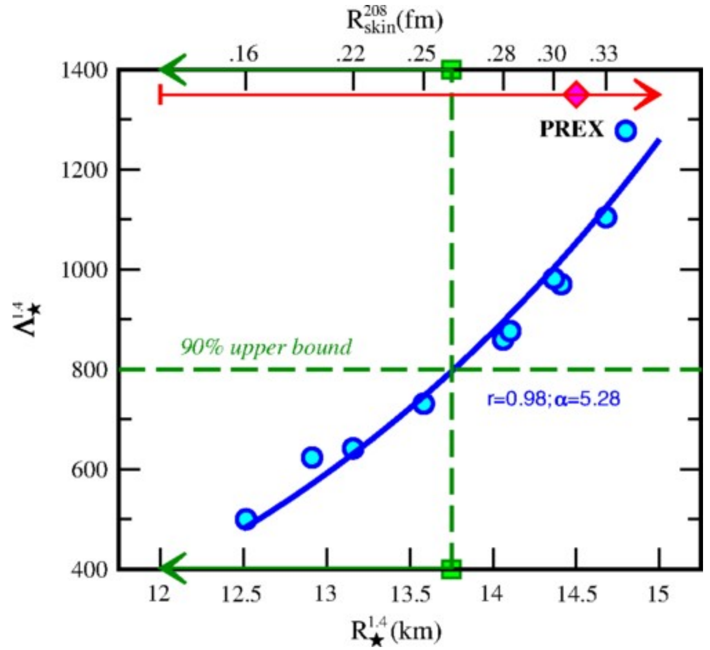
Figure 3 | Neutron star mass–radius diagram. The plot shows non-rotating

A two-solar-mass neutron star measured using Shapiro delay - P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts & J. W. T. Hessels - Nature volume 467, 1081–1083(2010)

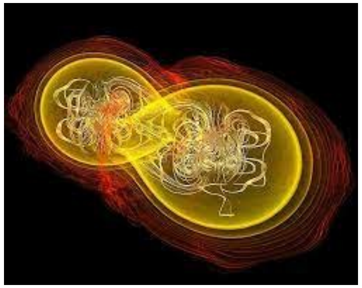
From Heaven: Gravitational wave signal from a binary neutron star merger

GW170817 from the binary neutron star merger → **constraint** neutron star **radius** and, thus, the **nuclear EoS**





Tidal deformability (Λ) is a quadrupole deformation inferred from **GW signal →** proportional to **restoring force.** Hence, sensitive to the **nuclear EoS**



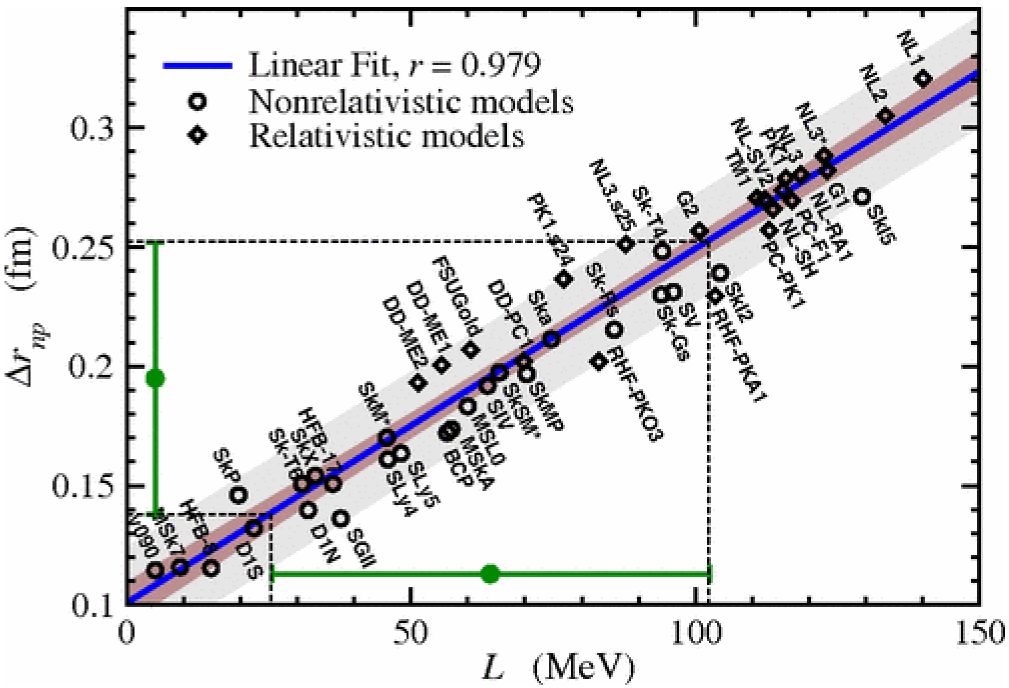
Neutron Skins and Neutron Stars in the Multimessenger Era
F. J. Fattoyev, J. Piekarewicz, and C. J. Horowitz Phys. Rev. Lett. 120, 172702 (2018)

From Heaven & Earth: neutron skin and the Radius of a Neutron Star

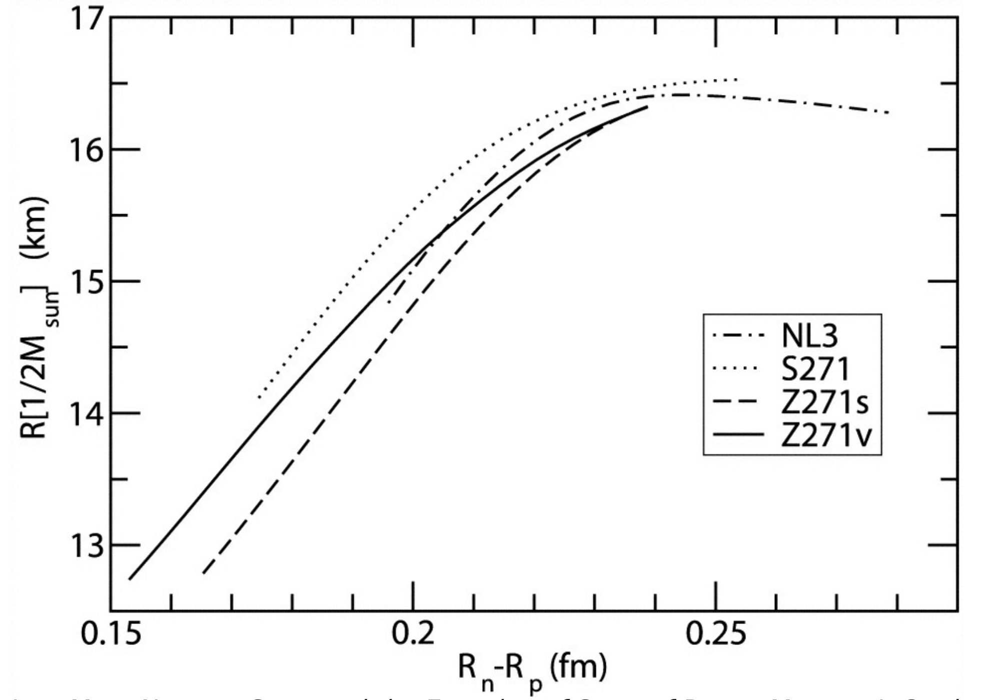
Both, the **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in neutron rich nuclei and the **radius** of a **neutron star** are related to the **neutron pressure** in infinite matter. The former around ρ_0 (L) while the latter at larger densities.

$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

→ For **small neutron stars**, that is, for small central densities: nuclear models predict a **linear** relation between **R** and $\boldsymbol{\Delta r_{np}}$



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



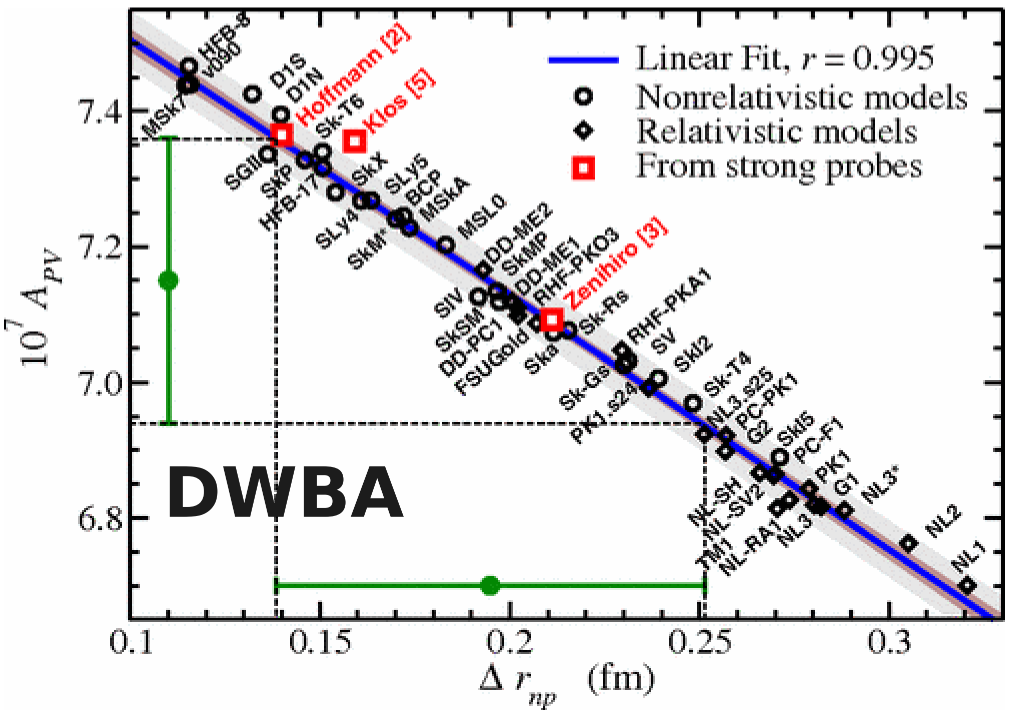
Low-Mass Neutron Stars and the Equation of State of Dense Matter - J. Carriere, C. J. Horowitz, and J. Piekarewicz - The Astrophysical Journal, 593 (2003) 463

From Earth: Parity violating electron scattering and the neutron skin

Polarized electron-Nucleus scattering:

→ In good approximation, **the weak interaction** probes the **neutron distribution** in nuclei while **Coulomb interaction** probes the **proton distribution**

→ **Different experimental efforts @ Jlab (USA) & MAMI (Germany)**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

→ **Electrons** interact by **exchanging** a **γ** (couples to **p**) or a **Z_0** boson (couples to **n**)

→ **Ultra-relativistic electrons**,depending on their helicity (±), will interact with the nucleus seeing a slightly different potential: **Coulomb** ± **Weak**

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

→ Main **unknown** is **ρ_n**

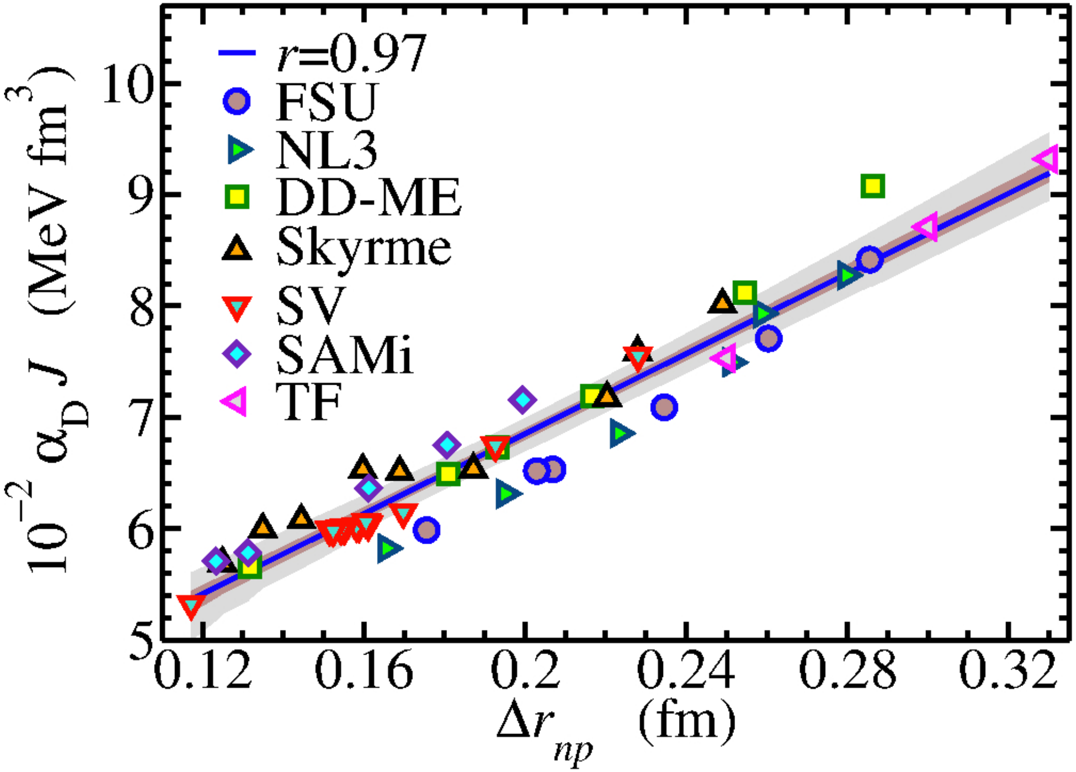
→ In **PWBA** for small momentum transfer **q**:

$$A_{pv} = \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 r_p^2}{3F_p(q)}\right)\Delta r_{np}$$

From Earth: dipole polarizability and neutron skin

The dipole **polarizability** measures the **tendency** of the nuclear **charge** distribution to be **distorted.**

From a macroscopic point of view **α ~ (electric dipole moment)/(external electric field)**



→ Using the **dielectric theorem**: the polarizability can be computed from the expectation value of the Hamiltonian in the constrained ground state H′=H+λD

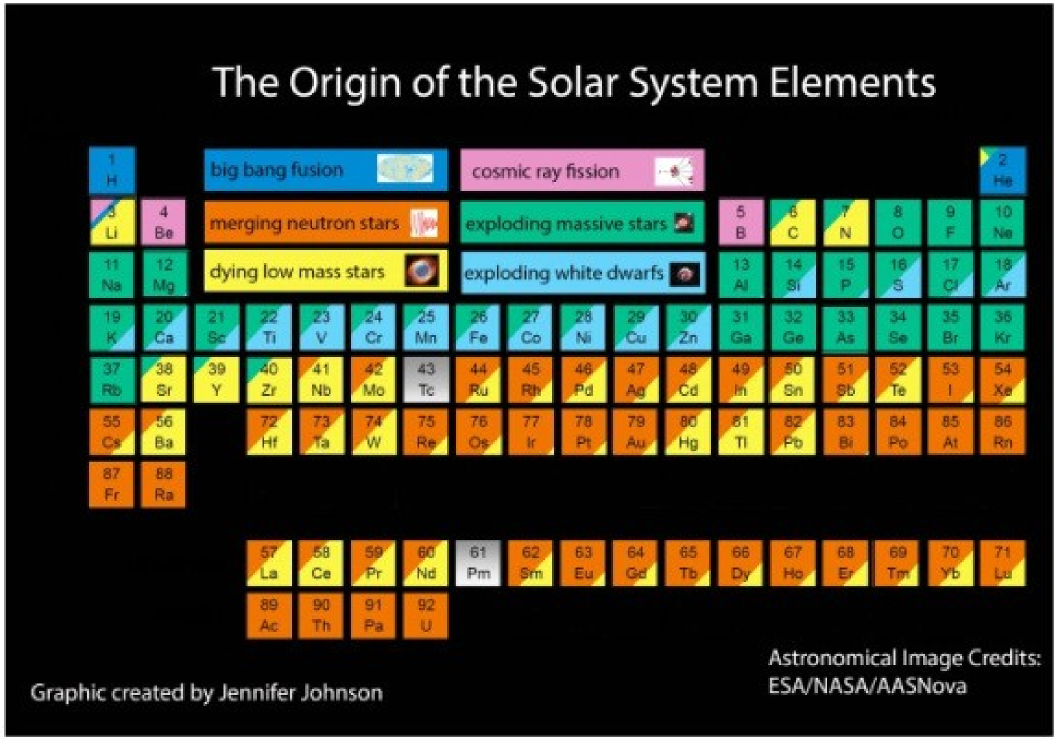
→ For guidance assuming the **Droplet model** for H, one would find:

$$\alpha_D \approx \frac{\pi e^2}{54} \frac{\langle r^2 \rangle}{J} A \left(1 + \frac{5}{2} \frac{\Delta r_{np} - \Delta r_{np}^{\mathrm{surf}} - \Delta r_{np}^{\mathrm{Coul}}}{\langle r^2 \rangle^{1/2} (I - I_{\mathrm{Coul}})} \right)$$

Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz

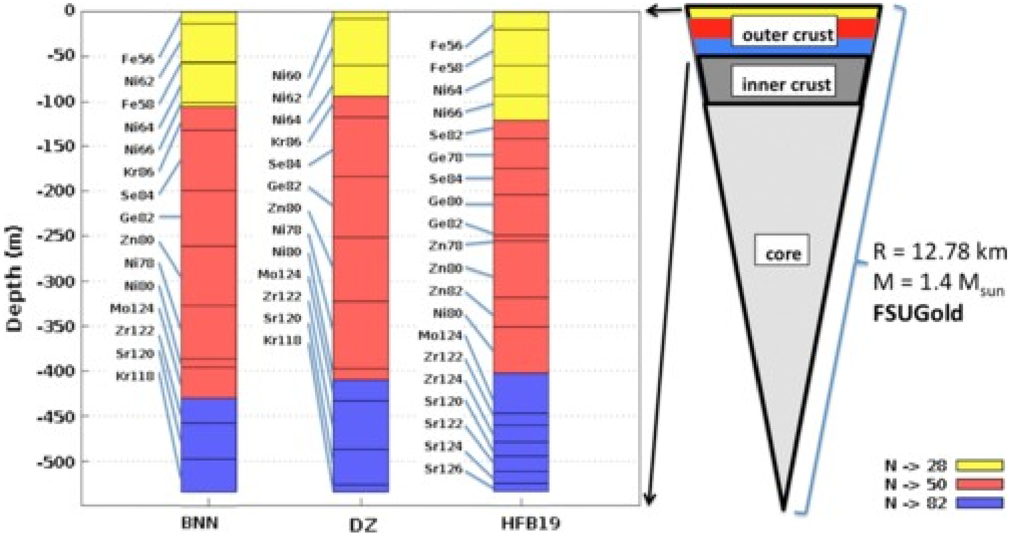
Phys. Rev. C 88, 024316 (2013)

From Heaven: Origin of elements



Binary neutron star merger produced about **10^{29}kg** of **heavy elements**!

The **crust** of a **NS** is made of very **exotic** **neutron rich nuclei,** stable only due to the extreme conditions (large densities).
Different nuclear models predict different compositions



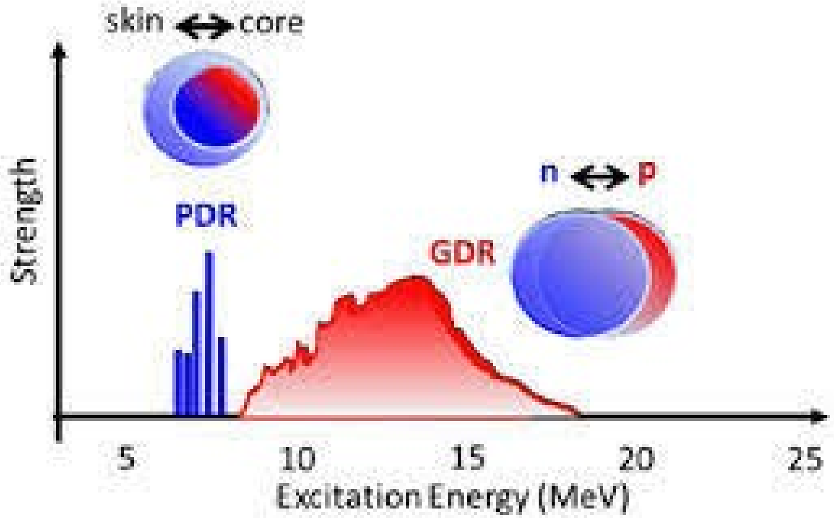
Nuclear mass predictions for the crustal composition of neutron stars: A Bayesian neural network approach R. Utama, J. Piekarewicz, and H. B. Prosper, Phys. Rev. C 93, 014311 (2016)

From Heaven & Earth: low energy dipole response and nucleosynthesis

The **largest** the **neutron pressure** among neutrons (~L), the more the **excess neutrons** (~skin) are ***"pushed out"*** in the **outermost** part of the **nucleus →** spatial ***decorrelation*** of some of those neutrons with the nucleons in the core produces **larger low lying** responses**.**

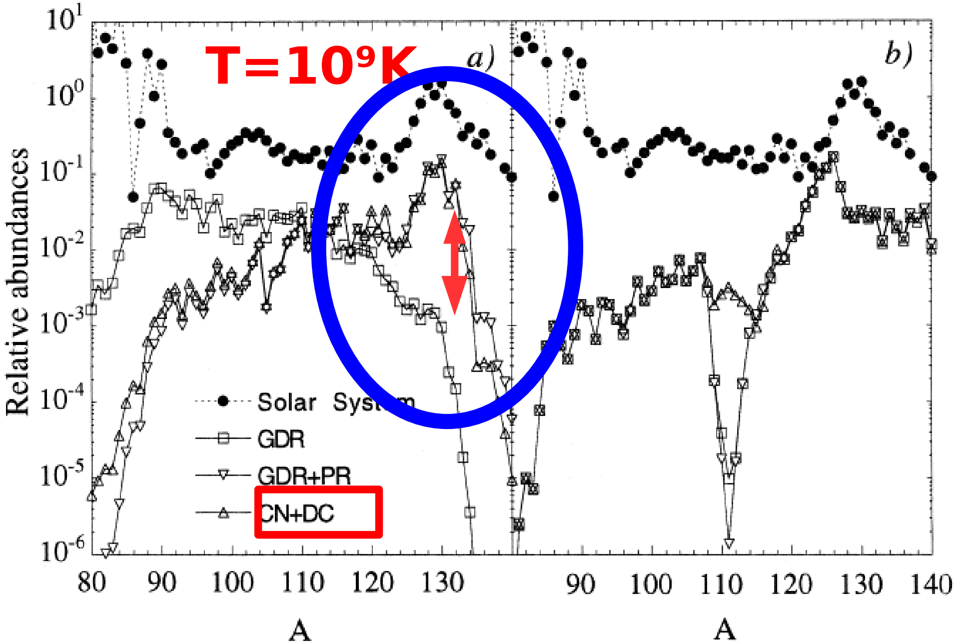
GDR=Giant Dipole Resonance

PDR= Pygmy Dipole Resonance



(nature of PDR still under debate)

Radiative neutron captures by neutron-rich nuclei and the r-process nucleosynthesis
S. Goriely, Phys.Lett.B 436 (1998) 10-18



Low energy dipole strength in neutron-rich nuclei influences the neutron capture cross section and, thus, the r-process nucleosynthesis

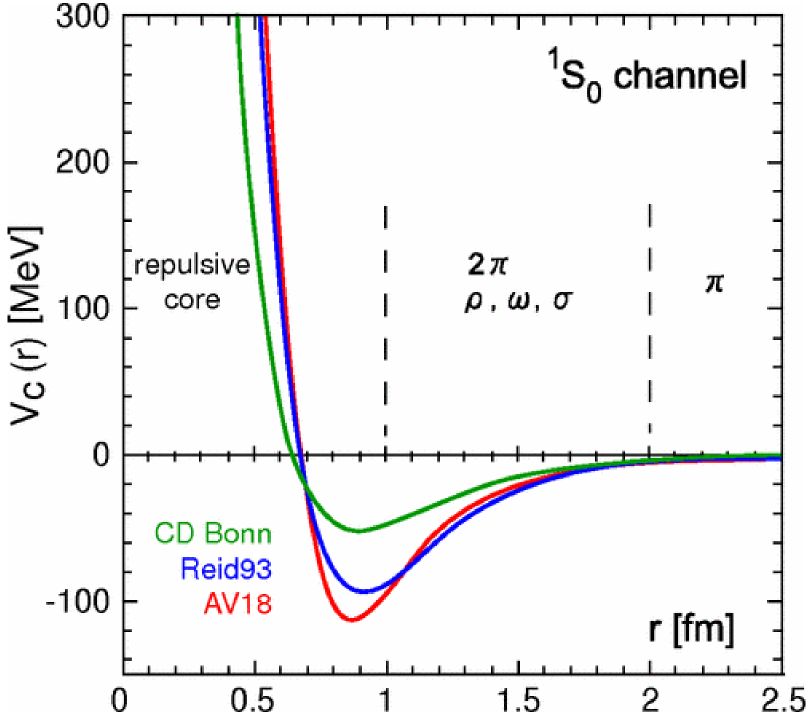
How are we dealing with the nuclear many-body problem?

(brief discussion)

Nuclear Many-Body Problem

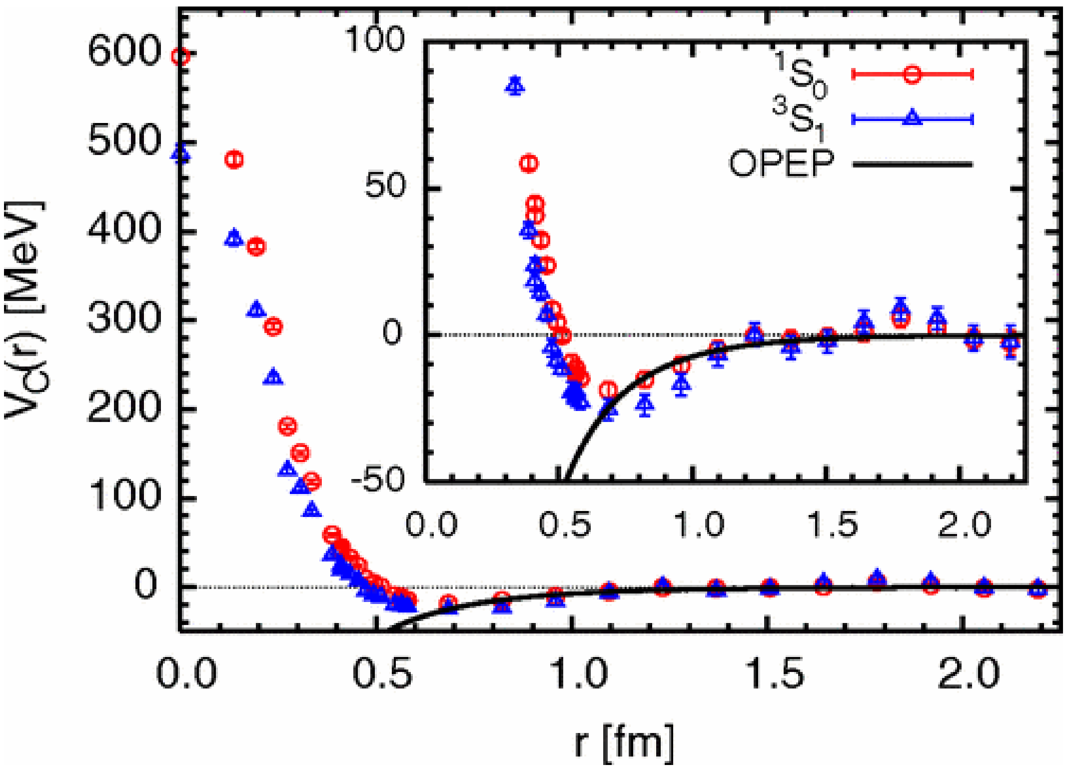
Underlying interaction: the “so called” **residual strong interaction** = **nuclear force** has **not** been **derived yet** (with the precision needed) from first principles as **QCD** is **non-perturbative** at the **low-energies** relevant for the description of nuclei.

Phenomenological



Nuclear Force from Lattice QCD - N. Ishii, S. Aoki, and T. Hatsuda
Phys. Rev. Lett. 99, 022001 (2007)

Lattice QCD (m_π/m_ρ~0.6)



Hohenberg-Kohn theorems

P.Hohenberg, W. Kohn, Phys. Rev. 136, B864 (1964)

→ Assuming a system of **interacting fermions** in a confining **external potential**, there exist a **universal** functional **F[ρ]** of the fermion density **ρ**:

$$E[\rho] = \langle \Psi | T + V + V_{\text{ext}} | \Psi \rangle = F[\rho] + \int V_{\text{ext}}(r)\rho(r) d\vec{r}$$

→ and it can be shown that

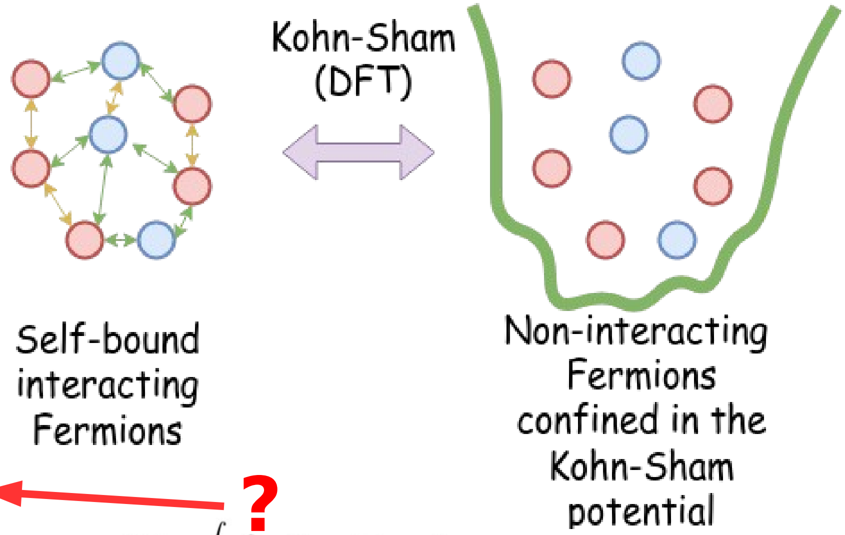
$$\min_{\Psi} \langle \Psi | T + V + V_{\text{ext}} | \Psi \rangle = \min_{\rho} E[\rho]$$

so **E[ρ]** has a **minimum** for the **exact ground-state density** where it assumes the **exact energy** as a value.

Kohn-Sham realization
$F[\rho] \rightarrow T_{\text{non-int.}}[\rho] + V_{KS}[\rho]$

For **any interacting system**, there exists a **<u>local</u> single-particle potential $V_{ks}(r)$**, such that the **exact ground-state** density of the **interacting system** equals the **ground-state density** of the **auxiliary non-interacting system**:

$$\rho_{\text{exact}}(\vec{r}) = \rho_{\text{KS}}(\vec{r}) = \sum_{i=1}^{A} |\phi(\vec{r})|^2$$



where φ are single-particle orbitals and the total wave-function correspond to a Slater determinant. The **E[ρ] is unique**

$$E[\rho] = T[\rho] + \int V_{\text{KS}}(\vec{r})\rho(\vec{r})d\vec{r}$$

?

$$F[\rho] = \int d^3r \; f(\mathbf{r}, \rho, \nabla\rho, \ldots).$$

where **T[ρ]** is the **kinetic energy of the non-interacting system** and for which the variational equation

$$0 = \frac{\delta E[\rho]}{\delta \rho} = \frac{\delta T[\rho]}{\delta \rho} + V_{\text{KS}}$$

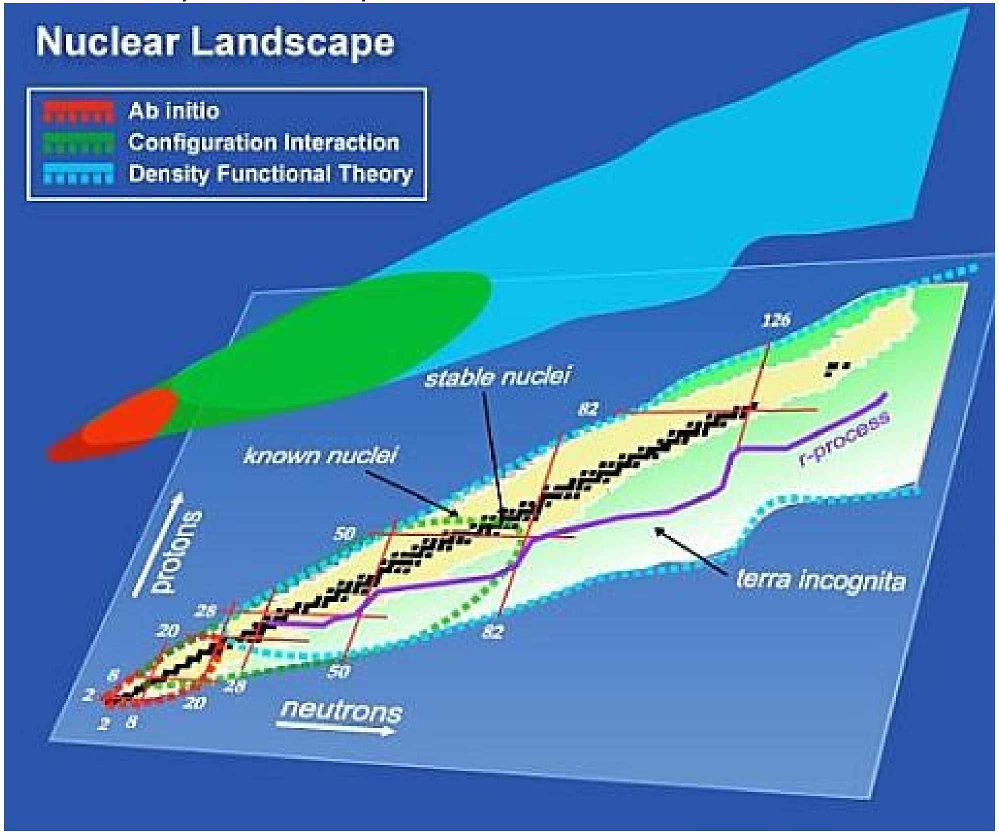
DFT Euler equation:

$$\frac{\delta T[\rho]}{\delta \rho(\mathbf{r})} + v_{KS}([\rho], \mathbf{r}) = \mu[\rho]$$

yields to the **exact ground state density and energy**

Advantadges and disadvantages of DFT

UNEDF http://unedf.mps.ohio-state.edu/



→ **ADVANTAGES OF DFT:**

• **exact theory** that can be applied to the **whole nuclear chart**

• **many-body** problem mapped onto a **one-body** problem without the need of explicitly involving inter-nucleon interactions!!! (computational cost and interpretation of observables in terms of single-particle properties)

• **HK generalised in (almost all) possible ways**: time dependence, degenerate ground-state, magnetic systems, finite T, relativistic case ...

• **any one body observable is within the DFT framework** (this includes also some sum rules related to nuclear excitations)

→ **DISADVANTAGES OF DFT:**

• various **proofs of HK theorems** do **not** give any clue on **how to build the functional**.

• **no** direct **connection** with **realistic NN or NNN interaction** if current approaches to EDF are not improved (some attempts already exist)

• **no systematic** way of **improvement** (evaluate syst. Errors) so far.

Avenues to improve EDFs? (@Milano)

→ We are working in two main directions:

1) **Inverse Kohn-Sham problem**: determine the V_{KS} and then E[ρ,...] from experimental and/or ab initio density distributions. With different **Bachelor and Master Thesis** students

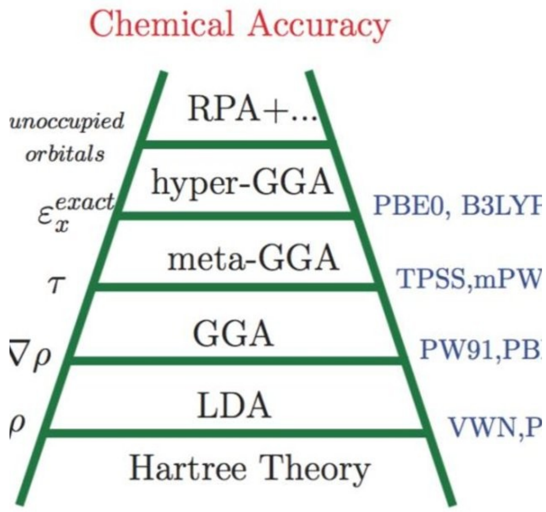
First step in the nuclear inverse Kohn-Sham problem: From densities to potentials

G. Accorto, P. Brandolini, F. Marino, A. Porro, A. Scalesi, G. Colò, X. Roca-Maza, and E. Vigezzi
Phys. Rev. C **101**, 024315 – Published 28 February 2020

2) **Mimic** strategy (**Jacob's Ladder**) in **many-electron systems** to systematically improve nuclear EDFs without using *phenomenological* parameters (as long as possible). With one **PhD** (Francesco Marino) and hopefully one postdoc in the future.

Nuclear energy density functionals grounded in *ab initio* calculations

F. Marino, C. Barbieri, A. Carbone, G. Colò, A. Lovato, F. Pederiva, X. Roca-Maza, and E. Vigezzi
Phys. Rev. C **104**, 024315 – Published 9 August 2021



Testing and understanding many-body theories

→ **Harmonic potential theorem:** should be fulfilled by any meaningful many-body theory. With **Bachelor** student and **PostDoc**.

Harmonic Potential Theorem: Extension to Spin-, Velocity-, and Density-Dependent Interactions

S. Zanoli, X. Roca-Maza, G. Colò, and Shihang Shen (申时行)

Phys. Rev. Lett. **123**, 112501 – Published 13 September 2019

→ **Exactly solvable models:** help in better understanding the involved approximations. With **Master** student and **PostDoc**.

LETTER

Extended Lipkin–Meshkov–Glick Hamiltonian

R Romano[1], X Roca-Maza[3,1,2], G Colò[1,2] and Shihang Shen(申时行)[1,2]

Published 8 April 2021 • © 2021 IOP Publishing Ltd

Journal of Physics G: Nuclear and Particle Physics, Volume 48, Number 5

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→ Hiroyuki **Sagawa** (University of Aizu & RIKEN)

→ Shihang **Shen** (Forschungszentrum Jülich)

→ Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)

→ Jorge **Piekarewicz** (Florida State University)

→ Nils **Paar** (University of Zagreb)

→ P.-G. **Reinhard** (University of Erlangen-Nürnberg)

→ Misha **Gorchtein** & Oleksandr **Koshchii** (Johannes Gutenberg-Universität)

→ Chuck **Horowitz** (Indiana University)

→ Haozhao **Liang** (University of Tokyo)

→ Witold **Nazarewicz** (FRIB and Michigan State University)